Celebrating Mathematics

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This is the year we celebrate American mathematics.

One hundred years ago, Thomas Scott Fiske founded the New York Mathematical Society, precursor to the American Mathematical Society. Then, mathematics flourished in Europe but barely existed in the New World. Today, the American Mathematical Society sustains the world's strongest environment for research mathematics. It has given science—and society—much to celebrate.

The Mathematical Association of America is nearing its 75th anniversary as an organization devoted to the teaching of mathematics, especially at the collegiate level. Ever since 1915, the Society and the Association have cooperated on many jointly sponsored activities. Our missions—mathematical research and mathematics teaching—are like braided strands that together form a strong fiber for the fabric of American science.

On behalf of the Mathematical Association of America, I salute our sister society for a century of accomplishment, for the creation of a rich tapestry of beautiful and useful mathematics. The AMS centenary beckons all of us in the mathematical community to look around to our colleagues, to look outward to society, and to look ahead to the future of mathematics. We join the celebration, to applaud the achievements of American mathematics; to proclaim that strong mathematics contributes to strong science, to strong defense, and to a strong economy; and to challenge American youth with the excitement of mathematical discovery.

Of course mathematics did not begin with the founding of the American Mathematical Society. Two hundred years before that—three hundred years ago—Newton published *Principia Mathematica*, thereby establishing mathematics as the methodological paradigm of theoretical science. From this paradigm have emerged many mathematical sciences, and many mathematics societies. The American Statistical Association celebrates its 150th birthday in 1989; the Society for Industrial and Applied Mathematics celebrated its 35th Anniversary last year, at the same time that the Association for Computing Machinery celebrated its 40th birthday.

So "100 Years of American Mathematics" is a bit of hyperbole, a slogan that throws images beyond literal meaning. It is in part a strategy to focus attention on the need for sustained support for mathematics. We celebrate 100 Years of American Mathematics not because we equate Thomas Fiske with Isaac Newton, but because it is a timely device to set important issues before the mathematical world, the scientific community, and the attentive public. Despite a century of success, there are urgent matters that need attention.

Mathematics Today

Those of us who are part of the mathematical community recognize the enormous current vitality of the mathematical sciences. Applications, enthusiasm, initiatives, opportunities, and unity are the "vowels" around which mathematical language is formed, a language in which mathematicians express solutions to old problems and explore new areas of fruitful growth. Explosive growth is a sign of remarkable health, but it does leave the thousands of us who try to keep up panting breathlessly as the research leaders disappear over the horizon.

Dozens of areas of active research could be cited to document the vitality of contemporary mathematics (see [**17**], [**18**], [**24**]). For some of these, look at the program of this meeting. Better still, look at the whole program for 100 Years of American Mathematics, especially at the marvelous symposium that is part of the 1988 AAAS meeting in Boston and at the special series of expository lectures on contemporary mathematics that is featured at the AMS Centennial Meeting in Providence.

Today I want to highlight four areas as examples of the unity and applicability of mathematical research: computational statistics, mathematical biology, geometrical mathematics, and nonlinear dynamics. Each of these offers opportunities for initiatives in mathematical research and for engendering enthusiasm among students. Think especially about the latter, about opportunities for initiatives in mathematics education, while I briefly outline each of these four areas.

Computational Statistics. The statistical sciences study problems associated with uncertainty in the collection, analysis, and interpretation of data. Not surprisingly, the increasing use of computers to record and transform data has generated a host of new challenges for the statistical sciences.

For example, analysis of data from electronic scanning devices (in tomography, in aircraft or satellite reconnaissance, in environmental monitoring) has produced an urgent need for statistical analysis of data that has an inherent spatial structure [**26**]. Research in this emerging field of spatial statistics employs a wide variety of mathematical, statistical, and computational techniques: problems of separating signals from noise borrow ideas from engineering; ill-posed scattering problems employ methods of numerical linear algebra; and smoothing of data requires statistical techniques of regularization. Underlying all this is the inherent geometry of the problem, which in many cases is dynamic and non-linear.

Many applications of statistics (for example, to clinical data from innovative medical protocols) involve small data sets from which one would like to infer meaningful patterns. Bradley Efron and others have pioneered innovative, computationally-intensive statistical techniques that use the limited available data to generate more data with the same statistical characteristics ([**4**], [**5**], [**14**]). By resampling the given data repeatedly, these so-called bootstrap methods generate millions of similar possible data sets which yield accurate approximations to various complex statistics. By comparing the value of statistics for the given sample with the distribution obtained by these resampling schemes, one can determine whether the observed values are significant.

Mathematical Biology. Nothing better illustrates the potential for mathematics in the biological sciences than the many traces of mathematics behind the Nobel prizes. For example, the 1979 Nobel Prize in medicine was awarded to Allan Cormack for his application of the Radon transform to the development of tomography and CAT scanners. The 1984 Nobel Prize in chemistry was awarded to biophysicist Herbert Hauptman, President of the Medical Foundation of Buffalo, for fundamental work in Fourier analysis pertaining to X-ray crystallography [**19**].

Indeed, recent research in the mathematical sciences suggests dramatically increased potential for fundamental advances in the life sciences using methods that depend heavily on mathematical and computer models. Structural biologists have become genetic engineers, capturing the geometry of complex macromolecules in supercomputers and then simulating interaction with other molecules in their search for biologically active agents. Using these computational methods, biologists can portray on a computer screen the geometry of a cold virus—an intricate polyhedral shape of uncommon beauty and fascinating geometric features—and search its surface for molecular footholds on which to secure their biological assault.

Geneticists are beginning the monumental effort to map the entire human genome, an enterprise requiring expertise in statistics, combinatorics, artificial intelligence, and data management to organize billions of bits of information. Ecologists—the first mathematical biologists—continue to use the extensive theories of population dynamics to predict the behavior and interaction of species ([**9**], [**32**]). Neurologists now use the theory of graphs to model networks of nerves in the body and the neural tangle in the brain [**10**].

Cell biologists study the replication of DNA using the newly discovered algebraic classification of knots ([**12**], [**13**]). Epidemiologists monitor the spread of AIDS with techniques that blend innovative statistics with classical analysis. And, finally, physiologists employ contemporary algorithms applied to nineteenth-century equations of fluid dynamics to determine such things as the effects of turbulence in the blood caused by cholesterol or swollen heart valves [**11**].

Geometrical Mathematics. Ever since Euclid, geometry has been one of the major pillars of core mathematics. After decades of decline (especially in mathematics teaching), the geometrical view in mathematics has undergone a renaissance, assisted both by the development of new theoretical tools and by the power of computer-based visual representation. In a very real sense, geometry is once again playing a central role on the stage of mathematics, much as it did in the Greek period.

Geometry claimed two of the three 1986 Fields Medals, which were awarded to Michael Freedman and Simon Donaldson for work in the geometry of four dimensional manifolds ([**1**], [**3**], [**21**]). By exploiting properties of the Yang-Mills field equations that reflect the wave-particle duality of matter, Donaldson showed that the differential geometry of four dimensional manifolds was vastly different than that suggested by their topological structure. Freedman provided the topological classification. Together, their work yielded not only deep understanding of four-dimensional manifolds, but the surprising insight that in four dimensions there are differentiable manifolds that are topologically but not differentiably equivalent to the standard Euclidean four dimensional space. Already, insight from this work has led to applications in string theory—the new super-symmetric theory of elementary particles—thereby providing fresh evidence (see [**34**]) of what Eugene Wigner called the "unreasonable effectiveness" of mathematics in the physical sciences.

Computer graphics provides a powerful new tool that extends geometrical techniques into many parts of mathematics. Computers—especially supercomputers—can calculate and display various mathematical structures in visual form, thereby enabling mathematicians to "see" the significance of abstract patterns that before could be interpreted only by formal means. For example, visual representations of solutions of differential equations often produce conjectures that open up whole new insights into the behavior of the system which the equations represent. Geometrical studies themselves regularly yield innovative challenges in the design of new algorithms and data structures, with spin-off benefits to applications in computer science (for example, database systems and word processing) far removed from the original geometric problem. The newly launched Geometry Supercomputing Project at the University of Minnesota is one example of the growing interaction of researchers in geometry with those in theoretical computer science.

Nonlinear Dynamics. Only in recent years have we been able to provide mathematical analyses of problems that are essentially nonlinear (for example, turbulence in fluids). These have been made possible by novel analytical methods, clever numerical simulation, and visual display on computer screens. Applications range from airfoil design to plasma physics, from oil recovery to studies of combustion ([**7**], [**15**], [**16**], [**22**]).

Nonlinear dynamics has yielded many surprises (see [**6**], [**8**], [**33**]), including long-term localized structures (e.g., the Red Spot on Jupiter), deterministic (rather than stochastically) generated chaotic motion (typical of some weather phenomena), and fractal patterns at the interface between fluids (for example, displacement of oil by water). The mathematics of nonlinear dynamics involves a great deal of traditional analysis (especially differential equations), reinforced by iterative processes, automata theory, and fractal geometry.

Computer display of nonlinear phenomena makes visible patterns that would never have been noticed by analytic means alone. In research on dynamical systems, on the transition from order to chaos, and on the emergence of fractal shapes from smooth flows, computers are to mathematics what telescopes and microscopes are to science: they increase by a thousandfold the portfolio of patterns that mathematicians can see and investigate.

The Newtonian revolution not only established mathematics as a paradigm for scientific reasoning, but it also established determinism as a paradigm for the behavior of physical systems. Nonlinear dynamics—a direct descendant of Newtonian mathematics—shows how ambiguity and uncertainty can arise in even simple deterministic systems, and how the onset of chaos itself can be predictable. In its power to change our Newtonian view of mathematics, nonlinear dynamics is as revolutionary as quantum mechanics: each breaks the bond of determinism and reveals entirely new structures that often defy what we have come to think of as common sense.

Mathematics in the Classroom

I choose these examples, drawn from diverse areas of the mathematical sciences, not just to illustrate the vitality of the mathematics that we are here to celebrate, but to provide a mathematical and intellectual backdrop for what is, unfortunately, a very different portrait of mathematics in the classroom.

In research we see a lot of geometry, a lot of data, a lot of science, a lot of computation—together with more traditional mathematical tools. We see investigation, exploration, and a continual search for pattern. Contemporary mathematics compels attention. It has the power to excite the best minds of our youth and to stimulate renewed creativity in teaching mathematics.

But this mathematics is *not* the mathematics taught in typical school or college classrooms. Far too often, mathematics in the classroom is a freezedried mathematics—rigid, cold, and unappealing. Instead of exploration there is drill; instead of investigation, imitation. From elementary school arithmetic to college calculus, mathematics in the classroom is dramatically different from mathematics in practice.

You've all heard the litany of problems with school mathematics. It ranges from poor test performance on international assessments to declining interest among Americans in pursuing advanced study of mathematics. I won't repeat this evidence here since it has been widely publicized in reports, journals, and newsletters ([**20**], [**30**], [**31**]).

Not yet so well known are the current attempts by several organizations (for example, NCTM, MSEB, AAAS, and the University of Chicago) to reverse this decline ([**23**], [**27**], [**28**]). These projects have engaged school teachers, mathematics educators, and mathematics researchers in collaborative work on the problems of mathematics education in the schools. Although these projects differ greatly in purpose and detail, their emerging recommendations have much in common that resonates with the nature and practice of contemporary mathematics:

- Mathematics should be taught in a natural context;
- Students should be encouraged to create, to invent, and to participate;
- Calculators and computers should be used throughout the mathematics curriculum;
- New topics (for example, algorithms, data analysis, estimation) should be introduced into the mainstream curriculum;
- Facility in computation need not be a prerequisite to the study of mathematics;
- Mathematics should be studied as an integrated whole;
- Mathematics should help build students' abilities to reason logically;
- Communication is an important goal of mathematics instruction.

It doesn't take much imagination for someone who is familiar with examples of contemporary mathematical science to see how student involvement in such mathematics could contribute to achieving these goals. Just the examples I have cited—statistics, biology, geometry, dynamics—overflow with natural context and with opportunities for students to use computers to discover patterns. These examples reveal far better than the isolated morsels of the traditional curriculum that new mathematical methods are needed to solve new problems; that communication is important for one to just understand, let alone express, the subtleties revealed by mathematical analysis; and that mathematics in action requires not only calculation and logic, but also intuition, imagination, and organization.

Unfortunately, too few of those who are most knowledgeable about mathematical research are working with teachers to translate their research into experiences suitable for classroom exploration. And far too few teachers—even at the college and university level—have the background, the interest, or the time to learn enough about modern mathematical research to translate it successfully into classroom experiences. So long as a large gap remains between those who create mathematics and those who teach mathematics, we cannot expect students to see in mathematics the challenge of an exciting and intellectually rewarding career.

Among the anniversaries we celebrate this winter is the centenary of the birth of George Pólya, who died just three years ago. Pólya was one of the rare mathematicians who made major contributions both to mathematics research and mathematics education. Andrei Kolmogorov, who died in October, was another.

The December 1987 issue of *Mathematics Magazine* is devoted to Pólya's life and work; I urge you to read it—I'm sure you'll find it as fascinating as I did. In that issue, Alan Schoenfeld wrote an interesting analysis of Pólya's theory of heuristics and its impact on teaching students to solve mathematical problems. Schoenfeld begins with Pólya's dictum that a good mathematics education is one that provides systematic opportunities for students to *discover* things.

How often does our teaching really do that? Think about the contrast of the stylized two-column proofs of high school geometry with the exploratory possibilities of three-dimensional computer graphics, or of the linking of knots, or of topological transformations of common surfaces. Or think, as many did at the NRC colloquium on Calculus for a New Century, about the contrast between the five thousand exercises in typical calculus books that mostly ask students to imitate calculator buttons, and the discovery potential in symbolic computer systems or in visual presentation of nonlinear dynamics.

Pólya's discovery dictum was echoed (perhaps unconsciously) at the calculus colloquium by Oberlin College President Frederick Starr [**29**]. He cited research [**2**] on college student career choices that shows "incontrovertibly" that the only institutions that are successfully resisting the precipitous decline in the percentage of students entering careers in science are those that base their pedagogy on a kind of apprenticeship system. In these schools students are brought into the laboratory to pursue real science under the direct guidance of professors who are themselves actively engaged in the scientific quest.

Traditionally, it has been the laboratory sciences—notably chemistry—that excel at attracting students by a style of education that involves students in the discovery of science. But now mathematics can do the same. With frontiers as exciting as chaotic systems and spatial statistics, there is no longer any reason for mathematics to fall behind the more glamorous laboratory sciences in attracting the interest and enthusiasm of our brightest youth.

Causes for Celebration

In celebrating mathematics, we point to the immense success of mathematical research in creating an intellectual understanding of space and number, of order and chaos, of pattern and disarray. Mathematics itself is beautiful, powerful, and deep; the process of doing mathematics is personally stimulating and intellectually rewarding.

Nevertheless, the profession of mathematics—as distinct from the discipline of mathematics—is not in good health. Decades of neglect in maintaining clear communication channels—with education, with science, with the public—have left mathematics isolated from the support systems that are vital to its health and well-being.

Our celebration must become a commitment to communicate. As we move into the second century of American mathematics—to continue the hyperbole—we should build on the impressive accomplishments in mathematics itself to bring the excitement and power of the mathematical sciences to all Americans. Here are five causes to champion as we celebrate mathematics in 1988.

▪ INVOLVE STUDENTS IN THE PRACTICE OF MATHEMATICS

The evidence is overwhelming that students receive a better education, and are more likely to be attracted to mathematics, when they are actively involved in mathematical experiences [**25**]. Pólya called it discovery learning; Starr described it as apprenticeship education. Although only a few mathematics students in the United States now receive the benefit of this type of learning, there are many excellent models of such teaching: problems competitions, research experiences for students, exploratory computer graphics, innovative tutorials where students become teachers, and team-based internships in mathematical modelling. It is noteworthy that the National Science Foundation, under the research directorates, has once again begun to support programs that provide research experiences to undergraduates. Especially for undergraduate students, but also in appropriate degrees for younger students, we must tilt the balance of mathematics education towards greater student involvement in learning.

Doing this will be expensive, and might involve radical departures in the way we finance undergraduate and graduate education. Since teachers tend to teach as they were taught, the most effective way to promote discovery learning in the schools is to enhance apprenticeship learning in the colleges, where tomorrow's teachers are today learning how to teach by the examples set by their college professors. The time is ripe for department chairs to insist that universities fund teaching at a sufficient level that all undergraduates can be taught by experienced teachers who will involve students in the excitement of discovering mathematics.

Apprenticeship education will require significantly better integration both of research mathematics and of contemporary applications in the experience of students. Current research, new applications, and the emerging goals of mathematics education could resonate in ways that would greatly enhance student involvement in mathematical learning. But such resonance cannot happen so long as researchers and teachers continue to operate in separate spheres—worlds apart in mathematical outlook, experiences, and expectations. Resonance requires significant connection between stimulus and resonator. "Vertical integration" of mathematical knowledge that links research and applications with education, and that brings researchers into active contact with students and teachers, should become a major criterion in funding decisions that concern the support of research.

We should celebrate the wealth of interesting new mathematics by bringing this mathematics into every classroom in the nation. To do that will require changes in the way we judge teaching and in the way we judge research: each should be found wanting if it does not include appropriate linkage with the other. It is not enough that individuals be competent as teachers and separately as professionals: separate but equal is as inadequate as a model for the relation of teaching and professional activity as it is for racial composition of public schools. Our goal should be professional standards that insist on apprenticeship learning and vertical integration of mathematical research.

- Educate the Attentive Public

Far too many educated persons are ignorant of mathematics. More troublesome, most do not feel their mathematical ignorance to be a great handicap. Most successful lawyers, politicians, educators, business executives—and university administrators—have achieved positions of prominence with only a minimal (and frequently archaic) knowledge of mathematics. Moreover, many persons, whether well educated or not, harbor feelings of apprehension or even anxiety about mathematics due to an unpleasant early educational experience, often with something labelled the "new math." When we try to take the case for mathematics to the public—or even just to university administrators—we face not only the healthy skepticism that naturally greets any self-serving argument, but also ignorance, fear, and often hostility that is a legacy of our neglect of mathematics education.

The state of mathematics as a profession compels us to find ways to diminish public fear and ignorance of mathematics, for without broad public support—for teaching, for research, for encouragement of students—there is no possible way for the mathematical community on its own to sustain the momentum of the past half-century. Now, however, perhaps for the first time, the *breadth* of the mathematical landscape makes it possible at least to imagine overcoming this pervasive public apprehension of mathematics.

Mathematics now touches people's lives in ways that matter and that can be described and revealed in human terms. From symmetry and chaos to computers and cosmology, from AIDS epidemiology and nuclear risks to political polls and ozone depletion, mathematics lurks behind most manifestations of science and technology. The same mathematization of society that makes the task of public understanding so essential also provides the means by which the task can be started—by building on mathematical ideas that are part of daily experience.

Surely part of our celebration must be to tell the story of mathematics to the attentive public. We are not publicists, but we are teachers. Suppose each department of mathematics made a commitment, just once each year, to arrange a public event that made mathematics visible in their community: an outside speaker who is working on something in which the public might be interested; a student project that involved a practical problem of interest to the community; a forum on the changing nature of school mathematics; or an exposition of a slice of mathematics related to some professor's research. Since the public is always more interested in people than in abstractions, there are, in addition, many good opportunities for news stories in hometown papers about the accomplishments of students. Someday some mathematics department should try to put out as many publicity releases on the accomplishments of their students as the athletic department does of theirs.

- Explore Fundamental Issues in Mathematics Education

Computers influence mathematicians not only by providing new tools for research and teaching, but also by posing deep questions about central issues in our discipline. Now that calculators can manipulate symbols and calculate answers,

- What—if not arithmetic—should be the core of elementary school mathematics?
- What—if not manipulation—should be the core of high school algebra?
- What—if not calculation—should be the core of calculus?
- What—if not calculus—should be the core of college mathematics?

At the same time that computers force attention on issues that are deeply rooted in unexamined tradition, mathematical research has transformed the nature of mathematics, opening up new options for what might be considered central and what derivative among the concepts of mathematics.

We need to find new threads of continuity with which to weave a mathematics curriculum for the twenty-first century. Finding appropriate central themes poses an immense challenge for the best minds among us, researchers and teachers alike. It gives common purpose to our diverse expertise, and sets a common agenda for those in research, those in college teaching, and those in school mathematics.

This provides yet another occasion for celebration: the opportunity, joined with the need, to transform school mathematics in ways that reflect the richness and diversity of mathematical research and mathematical applications.

Vertical integration of research, applications, and teaching will help bring about this transformation. But we need, in addition, structured opportunities for reflection in which the most synoptic thinkers among us bring their experiences in research, in applications, and in teaching to bear on the task of articulating central themes for mathematics education as we move into the next century.

▪ Ensure for All Students Equal Opportunities for Mathematical Success

Despite the culturally neutral status of mathematics (as compared, say, to biology), the last decade has produced distressing evidence of class and ethnic distinctions arising as a result of the way mathematics education is practiced in the United States. One-third of U.S. students—Blacks, Hispanics, and Native Americans—provides fewer than 10% of mathematics graduates, despite the evidence from isolated model programs that excellent retention and success rates can be achieved within a suitable educational context. Another third of U.S. students—white females—drops out of advanced degree programs in the mathematical sciences at twice the rate of male students.

Compounding these problems of class distinctions are the political, educational, and social side effects of large numbers of foreign-born teaching assistants in our major universities. It is easy to make a strong case for having many foreign graduate students in our universities; I join the many scientific leaders who defend this practice which has led to the United States being, in the words of Robert White, the "schoolhouse of the world." It is harder to make a case for placing inexperienced foreign graduate students in the classroom as instructors for American students who are not prepared to cope simultaneously with the challenge of a foreign culture, a foreign language, and a foreign discipline—namely, mathematics.

The consequence of these two unrelated trends is that both majority and minority students in college classes often receive mathematics instruction in a context that is culturally alien to them. Students who have too little in common with their teachers are unable to see themselves as future mathematicians or mathematics teachers. In this context, it is not surprising that even white U.S. males are no longer choosing careers in mathematical sciences. As I am sure you are well aware, the number of U.S. males receiving Ph.D. degrees each year in mathematics is less than 40% of what it was fifteen years ago.

The problem of opportunity in mathematics is so serious and so difficult that it is hard to even imagine a solution that is feasible, much less optimal. However, the mathematical community has an enormous resource that can be brought to bear on this problem—namely, the strong and culturally diverse community of research mathematicians that proves by its very existence the universality of mathematics. We need to find effective ways of conveying the rich and worldwide nature of mathematics to youth from the many subcultures that contribute to the American mosaic, and then to provide a context appropriate to their backgrounds in which to nurture mathematical talent.

The Professional Development Program at at Berkeley, led by Uri Treisman and Leon Henkin, has transformed the success rates of minority freshmen and enabled many to finish Berkeley and proceed to advanced or professional degrees. The University of Michigan has had considerable success in interesting minority students in careers in science through a special program that provides experiences in undergraduate research. These examples—and I'm sure there are others—show that it *is* possible to successfully attract talented minority students to careers in mathematics and science. Making progress in this endeavor would provide just cause for a true celebration.

- INVEST IN TODAY'S EDUCATION TO STRENGTHEN TOMORROW'S RESEARCH

Current debate about support for mathematics too often pits research against education, when in reality today's education is the pipeline for tomorrow's research. We read in the *Notices* of mathematicians who are under pressure to get research grants on pain of "being fired, having their teaching loads raised, or not getting raises." We hear continuing concern in the research community that, in times of limited budgets, new funds for education might be subtracted from the already limited amounts available for support of basic research—despite the fact that the percentage of federal support for science and mathematics that goes to education has slipped during the past four decades from nearly 50% to around 10%.

These arguments follow the same pattern that has led to the enormous growth in the federal deficit: by giving priority to the immediate needs of those in positions of power, we in effect support adults at the expense of children. Mathematicians, of all people, should be able to plan strategies that will optimize the strength of mathematical and scientific research over the long term. Part of that strategy is the recognition that education is not an alternative to research, but the foundation for future research.

Our celebration of the bounty of mathematical research must entail a commitment to education as the wellspring of research. We need to use multidimensional criteria in deciding on priorities for our community, seeking strategies that lead simultaneously to improvement in school mathematics, in collegiate mathematics, in graduate education, and in research.

All One System

Despite appearances to the contrary, mathematical research is inextricably entwined with mathematics education at all levels, with science and engineering, and with political, economic, and sociological aspects of society at large. Educators and researchers, teachers and professors, mathematicians and scientists—we are all part of a single system of knowledge on which contemporary society depends.

It is in this spirit that we join in celebrating the centenary of the American Mathematical Society. "100 Years of American Mathematics" provides the occasion for building new mathematical science on the firm foundation of the past century's research. Our centenary causes should be as broad and sweeping as our discipline:

- To involve students in the practice of mathematics.
- To educate the attentive public.
- To explore fundamental issues in mathematics education.
- To ensure for all students equal opportunity for mathematical success.
- To invest in today's education to strengthen tomorrow's research.

What transforms these causes from empty rhetoric to concrete options is the opportunity for education and communication implicit in the advances of today's mathematical sciences—in such areas as computational statistics, mathematical biology, geometrical mathematics, and nonlinear dynamics. It is in the frontiers of mathematical science—not in current textbooks or today's classrooms—that one can find the innovative and intellectually rewarding options needed to transform education, to excite our youth, to educate the public, and to reach all Americans.

This should be our centennial cause, not just for 1988 but for the rest of this century. It is a cause that can unite researchers and educators in a common challenge: To let the power and beauty of mathematics speak for itself.

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