A Conversation with Don Knuth: Part 2

Donald J. Albers and Lynn Arthur Steen

TYCMJ: *What was the inspiration to launch your series ("The Art of Computer Programming") in the first place?*

Knuth: *The Art of Computer Programming* developed when I was a second-year grad student at Cal Tech. I had been working as a private consultant writing compilers for different machines. In those days a software firm would ask for hundreds of thousands of dollars to write a compiler. But I didn't know that, and I had written one for $5,000. I guess the word got around that I knew how to write compilers. A little later, Richard Varga, who was an advisor to Addison-Wesley, suggested that they ask me to write a book about compilers. They came to me in January of '62 and said: "How would you like to write a book on writing compilers?" It occurred to me that I really did like to write. It sounded good; I decided that it would indeed be nice to write such a text.

I had gotten married in the summer of '61, and I wonder now how I broke the news to my wife that I was suddenly planning to write a book. Surely neither of us knew how much this was going to change our lives. Anyway that day I went home and sketched out 12 chapter titles that I thought would be nice, and then a little while later I signed a contract for a book about compilers. I got a chance to teach a course at Cal Tech during the fall of 1962, while still a graduate student, with the idea that the class notes would develop into the first three chapters.

Right after getting my Ph.D. in '63, I started to work hard preparing the chapter on sorting. I knew hardly anything about sorting, but I thought it would be nice to read up on the subject and to toss a chapter about sorting into a book about compilers, especially because the LARC Scientific Compiler had just come out and it was reputedly based on the idea of sorting the data in unusual ways. I found that sorting was really interesting, and pretty soon I found myself digging into lots of technical articles

The main thing that struck me was that the literature was so spotty. Computer science was a very new field, without an identity of its own, and standards of publication were not terribly high, especially when quantitative aspects of algorithm performance were concerned. A lot of the published articles were just wrong, so you had three possibilities: the wrong answer by the wrong method, the right answer by the wrong method, and the right answer by the right method. You had about a one-third chance on any of these possibilities. The literature on computing was already large but very unreliable, so it was clear even in 1962–63 that it would be nice to have a summary of the right parts of the literature. Publications were so bad, in fact, that people didn't even bother to read them, and the good ideas were being rediscovered because people found it easier to do this than to sort them out from the bad ones. Thus, one of my big motivations was to clean up the story that had been presented badly in the literature.

I guess I have an instinct for trying to organize things. At that time, everybody I knew who could write such a book summarizing what was known about computer programming had discovered quite a lot of the ideas themselves. It seemed to me that they would slant it to their own perspective, which would present only one side of the coin, their own particular part. By contrast, I really hadn't discovered anything new by myself at that point. I was just a good writer. That was very prominent in my mind: for example, I was the only computer scientist I knew who hadn't discovered how to compile arithmetic expressions by the precedence method. Ten people independently discovered it, yet the problem had baffled me; I hadn't seen my way through it. So I felt not only that the story needed to be presented, but that I could present it from a less biased viewpoint than these other people who seemed to have done their work more in isolation. I had this half-conceited and half-unconceited view that I could explain it more satisfactorily than the others because of my lack of bias. I didn't have any axes to grind but my own. (Then, of course, as I started to write things I naturally discovered one or two new things as I went, and now I am just as biased as anybody.)

But you asked about my original motivation. My original motivation was to write a text about how to write compilers, so I began drafting chapters. I was seriously planning to finish the book before my son was born. (He's a sophomore in high school now, so I'm currently trying to finish before he starts writing his own books!) I recently found copies of letters I wrote in 1965 saying, "I wish I could come to visit your university this summer, but I can't because I just have to finish this book-writing project." It was going to be just one book.

As I said, however, I started to get interested in sorting, after I had been writing for a while. There were so many interesting things on sorting. So the book was growing rapidly.

I eventually wrote a letter to Addison-Wesley saying: "Do you mind if this book is a little long? I would like to give a fairly complete presentation of the material." They said: "Don't worry, go right ahead, write whatever you want." So I kept gathering more and more stuff. In June, 1965, I had finally finished the first draft of the 12 chapters. It amounted to 3,000 hand-written pages of manuscript.

To me this was something of a longish book, yet only one volume. I thought I knew about books, and the printed letters in books seemed to be a lot smaller than the ones I write by hand. So I figured that about five pages of my handwriting would be about one page of book. Then I went ahead and did chapter one, typing it from the handwritten manuscript and sent it off to Addison-Wesley, just to see if it was okay. This was October, 1965. They hadn't heard from me for quite a while, and were wondering what was going on. I felt good, for at least I had finished chapter one. Incidentally, that chapter one was pretty much the same as the chapter that was eventually published.

Immediately I got back a letter from Addison-Wesley, from a person very high up in the company. It turned out he was one of the people who had originally talked to me in '62, but meanwhile he had been promoted three times. From the length of chapter one, the book was now estimated to be almost 2,000 printed

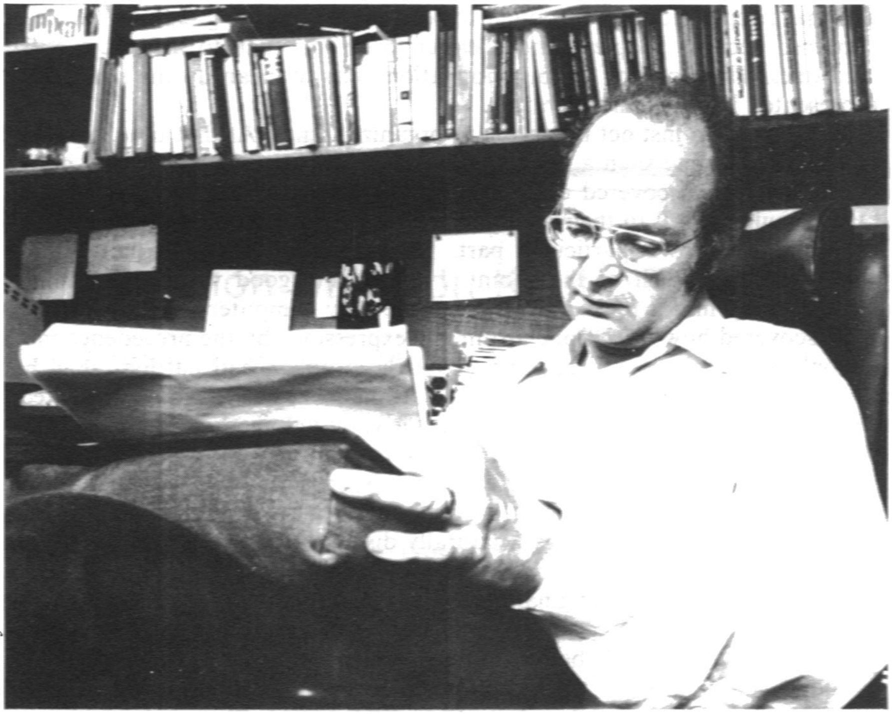

Photo by Tom Black

At work in his "book factory." Although Knuth is a skilled typist and attended secretarial school while in high school, he always prepares his books in longhand.

pages, and they said: "Don, you said you might be writing a longish book, but do you realize that one and one-half pages of typing is one page of book?" I thought to myself that it can't be so. "I've read a lot of books; these guys don't know what they're talking about." So I took one of their books, Thomas's *Calculus*, and I sat down at my typewriter and typed up a page of it. Lo and behold, they were absolutely right! Then I knew why it had taken me so long to get that first chapter finished—three and a half years to write the first chapter is not too good.

It gradually dawned on me how large a project this was going to be. If I had realized that at the beginning, I wouldn't have been foolish enough to start; I wouldn't have dared to tackle such a thing. But by 1965, of course, I was hooked because I still felt the need for these books, and I still felt the project ought to be done. I still believed that I was fairly unbiased and could try to be a spokesman for the people who were making the discoveries. I had collected so much material that I felt it was my duty to continue with the project even though it would take a lot longer than I had originally expected. I had done all of the background work and it would have been very hard to transfer it to anybody else.

Addison-Wesley took another look at the 12 chapters. At first it appeared that the material would fill up two volumes, then three volumes. Publishers have horror stories about ponderous tomes like that. So my guess is that they showed these things to some consultants, and the consultants recommended that seven of the 12 chapters would sell pretty well as individual volumes. So they suggested combining the remaining five chapters with the good seven, hoping to sell seven books this way. I think this was probably the motivation for the present plan. At any rate I saw that it was possible to reorganize the chapters so that they would fit together reasonably well in seven separate volumes.

Volume Two, for example, was the combination of my original idea for Chapter Two on random numbers and Chapter Six which was originally called "Miscellaneous Utility Routines." Suddenly I noticed that all but one of these miscellaneous utility routines was really about arithmetic. So I decided to call that chapter "Arithmetic." That little change in title suggested one or two sections that I ought to add; and as I added them, I came to a marvelous realization that there was this book out there waiting to be written, bringing together what is known about arithmetic from a computer scientist's point of view. As a result, that chapter almost wrote itself, and led me to fascinating things in a variety of journals that had just never been put together in book form. Having the title "Arithmetic," and having it bound in the second volume, is what turned out to add a lot of unity to the subject, a unity that probably hadn't been realized before. Most of the articles I found in the literature were written by people who were not aware of many of the other journals, nor of the relation between their ideas and others.

I got so excited about writing Volume Two that I started working day and night on it. As a result, I got a serious attack of ulcers, and had to change my whole life style. By the middle of Volume Two I kept thinking I was going to finish it soon, and I had something of a breakdown of my health in the summer of 1967. About the middle of Euclid's algorithm is where I broke down. That happened on what is now page 333 of Volume Two out of 688 pages. So I still had a lot to go at the time. I knew it, but I wouldn't admit it to myself.

I always underestimate time, too; otherwise, I never would have started writing these books. If only I had a better estimator of time! Because of my writing, I have now resolved not to give any lectures away from Stanford until 1990. All of book writing comes out of spare time. When I go on a trip to give a lecture, one day wipes out at least five days of book writing because when I come back I still have to do everything else that I was supposed to have done when I was gone. So the spare time disappears.

TYCMJ: *Your field, computer science, has been growing at great speed. Is it still possible to capture it? Is it growing faster than you can write?*

Knuth: Yes, perhaps. I'm thinking of the novel *Tristram Shandy*, a fictional autobiography whose supposed author goes through Volume One of his memoirs covering just the first year of his life. But I still believe it will be possible for me to finish. Volume Four, Combinatorial Algorithms, has exploded the most. Volume Four is the one that I'll return to immediately after I finish my typographical research. I think it is going to become Volumes 4A and 4B.

TYCMJ: *So it really is exploding?*

Knuth: In fact, the chapter called "Combinatorial Algorithms" that I planned on that first day in 1962 was thrown in almost as an afterthought. There were very few combinatorial algorithms at the time, but I liked that sort of programming so I thought it would be nifty to have a chapter about it. Almost none of the present material in Volume Four was known then. So when people talk about "combinatorial explosion" the words have a significant double meaning for me. At one point, three years ago, I think 70% of the journal articles being published in computer science were about combinatorial algorithms. Volume Four will surely be the hardest, because of the explosion.

TYCMJ: *Is the organization you worked out* 20 *years ago still pretty well holding its shape?*

Knuth: There are new topics, but I won't live long enough to include those. For example, I never promised to write about operating systems. Therefore, I'm very happy whenever I get a journal in the mail, if most of its articles are about operating systems.

TYCMJ: *How do you combine the discipline of writing with spontaneous creativity?*

Knuth: When I'm writing a book, I surround myself with that subject and nothing else. I read exhaustively on one area, and then after I finish that section, it goes out of my head, and I bring in another. That's what computer science calls "batch processing," as opposed to continual "swapping in and out" or "thrashing."

I don't read the literature as it happens. I only read the titles and abstracts to know where I can put articles on the agenda of things to read later. I do the same thing with quotes. I file the quote. Here is a wonderful one from the Beatles' songs: "There's nobody in my tree." That's just perfect for branch-and-bound methods. I hope to live long enough to finish just because I have so many quotes on file that are great.

When I'm working on a topic, I may have to read 60 papers on one subject. The first two I'll read slowly, but with the next 58 I know what to do already. When I read the first two, I use the strategy of trying to figure out the problems before I look at the answers. Then, I am ready for the vocabulary and ideas that are going to be occurring in the other papers.

I keep a little notebook, too; every day I write a summary of what I've worked on that day. It helps me to schedule myself a little bit and it helps me to realize how hard things are so that I can plan ahead. If too many days go by where I said I was just too tired and went to bed, or if it reads, "Today I goofed off," then it helps me to make a little more rational schedule.

For the seven-volume book project, I have to cross a threshold every day to get started when I am writing. I have to get psyched up for it. It is a long, on-going process; and I know that even after the end of the day I won't have finished. Every morning I wake up and say: "Another day, and the book isn't finished." I still feel a strong need for the book in the world, and that it is filling a necessary role, but all these logical arguments aren't going to make me get started. On the other hand, once I've started, then I'm excited about it, and it's hard for me to stop again. I have to force myself to stop and not just stay up all night. So I always read a variety of things—detective stories, or more serious works of fiction, or history, sort of rotating between them—at bedtime.

TYCMJ: *Do you do most of your writing at home?*

Knuth: Yes, always; it is not part of my Stanford job. It's all spare time.

TYCMJ: *Do you do it in long hand?*

Knuth: I can't compose at a typewriter. I can't even compose a letter to my relatives on a typewriter, even though I'm a good typist. I went to secretarial school during the summers of my high school years, and I learned to type 80 words a minute. I learned machine shorthand, and I learned Gregg shorthand. But I can't compose in any of those modes.

TYCMJ: *Why would a prospective physics major take those courses in high school?*

Knuth: I had summer jobs doing secretarial work, and I thought it would help me in college taking notes. But all I learned were the abbreviations for Dear Sir and Yours Very Truly, and that didn't help very much in my chemistry class. I kept making up new abbreviations, sitting in the back of the class with my stenograph machine; afterwards I couldn't figure out what I had put down, so I gave it up.

TYCMJ: *How did you come to write "Surreal Numbers"*?*

Knuth: I wrote it in one week, while on sabbatical in Oslo, Norway. It hasn't been a best seller, but it's been steady and translated into lots of languages. I'm glad for that. Writing *Surreal Numbers* was probably a once-in-a-lifetime experience for me. I got inspired to do it, and I guess there was a muse sitting behind me telling me what to write. The book just fell together, and I don't think I could do it again. That week was one of the most exciting periods of my life.

It was December, 1972, and I was in the midst of writing *The Art of Computer Programming*, when suddenly I got the idea for *Surreal Numbers* in the middle of the night. I woke my wife and I said, "Jill, you know how this series of seven volumes—the books I started on after we had been married for only six months—is affecting our lives? Well, it turns out that there's another book I would like to write too. But I don't think it will take me very long to finish this new one." I said that I thought I could write this other one in about a week, if I just worked on it and nothing else. To my great pleasure, she was also delighted by the idea. She said, "This is the best time in your life for you to do such a project."

We planned it so that after the new year I would get a hotel room in downtown Oslo near where Ibsen wrote his plays, so that I might be able to pick up some of the nuances of his art. Then I could work on this book and also she would come to meet me twice. (We always wanted to have an affair in a hotel room.)

During the three weeks or so before I started *Surreal Numbers*, as I would be walking along or skiing, I would be going through the first page or two of the book in my mind. But I didn't go any further than that, at the time, because I wanted the

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*In 1974, Addison-Wesley published a novelette by Don Knuth. Its title is *Surreal Numbers: How Two Ex-Students Turned on to Pure Mathematics and Found Total Happiness*. It contains a development of a remarkable new way to construct numbers. The new construction had been found by John Horton Conway of Cambridge University. One day over lunch in 1972, Conway briefly explained his system to Knuth. Knuth was so taken by this revolutionary approach that he was motivated to write a book about it. Martin Gardner says: "I believe it is the only time a major mathematical discovery has been published first in a work of fiction."

In a postscript to *Surreal Numbers*, Knuth explains his purpose in writing the book: " . . . my primary aim is not really to teach Conway's theory; it is to teach how one might go about developing such a theory. Therefore, as the two characters in this book gradually explore and build up Conway's number system, I have recorded their false starts and frustrations as well as their good ideas. I wanted to give a reasonably faithful portrayal of the important principles, techniques, joys, passions, and philosophy of mathematics, so I wrote the story as I was actually doing the research myself."

Don Knuth's first publication. "The Potrzebie System of Weights and Measures,"

SCIENCE DEPT.

When Milwaukee's Donald Knuth first presented his revolutionary system of weights and measures to the members of the *Wisconsin Academy of Science, Arts, and Letters*, they were astounded — mainly because Donald also has two heads. All kidding aside, Donald's system won first prize as the "most original presentation." So far, the system has been adopted in Tierra del Fuego, Afghanistan, and Southern Rhodesia. The U.N. is considering it for world adoption.

THE POTRZEBIE SYSTEM
OF WEIGHTS AND MEASURES

THE POTRZEBIE SYSTEM

This new system of measuring, which is destined to become the measuring system of the future, has decided improvements over the other systems now in use. It is based upon measurements taken 6-9-12 at the Physics Lab. of Milwaukee Lutheran High School, in Milwaukee, Wis., when the thickness of MAD Magazine #26 was determined to be 2.263348517438173216473 mm. This length is the basis for the entire system, and is called one potrzebie of length.

The Potrzebie has also been standardized at 3515.3502 wave lengths of the red line in the spectrum of cadmium. A partial table of the Potrzebie System, the measuring system of the future, is given below.

LENGTH

1 potrzebie = thickness of MAD #26
.000001 p = 1 farshimmelt potrzebie (fp)
1000 fp = 1 millipotrzebie (mp)
10 mp = 1 centipotrzebie (cp)
10 cp = 1 decipotrzebie (dp)
10 dp = 1 potrzebie (p)
10 p = 1 dekapotrzebie (Dp)
10 Dp = 1 hectopotrzebie (Hp)
10 Hp = 1 kilopotrzebie (Kp)
1000 Kp = 1 furshlugginer potrzebie (Fp)

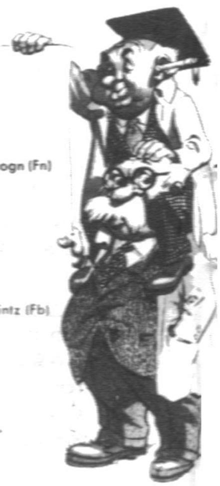
VOLUME

1 cubic dekapotrzebie = 1 ngogn (n)
.000001 n = 1 farshimmelt ngogn (fn)
1000 fn = 1 millingogn (mn)
10 mn = 1 centingogn (cn)
10 cn = 1 decingogn (dn)
10 dn = 1 ngogn (n)
10 n = 1 dekangogn (Dn)
10 Dn = 1 hectongogn (Hn)
10 Hn = 1 kilongogn (Kn)
1000 Kn = 1 furshlugginer ngogn (Fn)

MASS

1 ngogn of halavah* = 1 blintz (b)
.000001 b = 1 farshimmelt blintz (b)
1000 fb = 1 milliblintz (mb)
10 mb = 1 centiblintz (c)
10 cb = 1 deciblintz (db)
10 db = 1 blintz (b)
10 b = 1 dekablintz (Db)
10 Db = 1 hectoblintz (Hb)
10 Hb = 1 kiloblintz (Kb)
1000 Kb = 1 furshlugginer blintz (Fb)

*Halavah is a form of pie, and it has a specific gravity of 3.1416 and a specific heat of .31416.



appeared in MAD Magazine in June of 1957, when he was a high school senior.

TIME

1 average rotation of the earth = 1 clarke (cl)
.00001 cl = 1 wolverton (wl)
1000 wl = 1 kovac (kv)
100 kov = 1 martin (mn)
100 mn = 1 wood (wd)
10 wd = 1 clarke (cl)
10 cl = 1 mingo (mi)
10 mi = 1 cowznofski (cow)

DATE

October 1, 1952 is the day MAD was first published according to the old calendar. On new calendar, this is clarke 1 of cowznofski 1. Cowznofskis before this date are referred to as "Before MAD (B.M.)" and cowznofskis following this date are referred to as "Cowznofsko Madi (C.M.)" The calendar for each cowznofski contains 10 mingos named as follows: 1. Tales (Tal.) 2. Calculated (Cal.) 3. To (To) 4. Drive (Dri.) 5. You (You) 6. Humor (Hum.) 7. In (In) 8. A (A) 9. Jugular (Jug.) 10. Vein (Vei.)

FORCE (absolute)

a force which, when acting upon 1 blintz of mass for 1 kovac, causes it to attain a velocity of 1 potrzebie per kovac = 1 blintzal (b-al).
1000000 b-al = 1 furshlugginer blintzal (Fb-al)

ENERGY AND WORK

1 blintzal-potrzebie = 1 hoo (h)
1000000 h = 1 hah (hh)

POWER

1 hah per kovac = 1 whatmeworry (WMW)
1000 WMW = 1 kilowhatmeworry (KWMW)
100 KWMW = 1 aeolipower (A.P.)

RADIOACTIVITY

The quantity of radon which is in equilibrium with one blintz of radium = 1 hyme (how's your mom, ed)

HEAT ENERGY

The amount of heat necessary to raise one blintz of halavah 1° S = 1 vreeble (v.) 1000 v = 1 large vreeble (V)

TEMPERATURE

100° Smurdley (S) = optimum temperature for eating halavah (27°C.)

COUNTING

48 things = 1 MAD
49 things = 1 baker's MAD

ANGULAR MEASURE

100 quircits (‴) = 1 zorch (″)
100 zorch (″) = 1 zygo (§)
100 zygo (§) = 1 circle or circumference

MISCELLANEOUS MEASUREMENTS

1 light Cowz. = 1.14×10^{12} Fp.
1 vreeble = 574.8 hah
1 cosmo per sq. potrzebie = 3.1416 bumbles.
1 faraday = 122300 blobs/blintz equivalent weight.
1 electron-ech = 5.5794×10^{-16} hoo
1 blintz molecular wt. of a gas = 77.4 Cn

Density of H_2O = .31823 blintzes per ngogn (distilled water).
1 atmosphere pressure = $1.4532 \frac{b^2}{p}$
335.79 p of Hg.
1 b = 3234.4 b-al
gravity = 3234.4 p per kov. per kov.
speed of light = 114440 Fp. Kov.

BASIC CONVERSION FACTORS (ABBREVIATED TABLE)

1 inch = 11.222 potrzebie
1 mile = .71105 Fur-potrzebie
1 blintz = 36.42538631 grams
1 kiloblintz = 80.3048 pounds
1 Fur-blintz = 36.425 metric tons
1 ngogn = 11.59455 cc = .0125 qts = 2.3523 teaspoons

1 potrzebie = 2.2633 millimeters = .089106 inches
1 Furshlugginer potrzebie = 2.2633 Km = 1.4064 miles
1 [illegible] meter = 4.41[illegible] potrzebie
[illegible] = 44[illegible] Fur-potrzebie

1 wood = 144 minutes
1 kovac = 864 seconds
1 year = 3.6524 Cowznofskis
1 watt = [illegible] Whatmeworries
Horse Power = [illegible] Kilowhatmeworry
Aeolipower = [illegible] Horse Power
vreeble = [illegible]

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rest of the book to be fresh as it was being written. I wanted it to be a faithful recording of mathematical discoveries, so I didn't want to do any of the mathematics in advance. I only vaguely remembered what John Conway had told me at lunch a year before.

I got to the hotel and started to work. Fortunately, I didn't go through the scene you frequently see in the movies where the guy types the title of the book, stares at the page for awhile, and tears it up. I didn't have to go through that. I could write out the first page and most of the second, since I had that memorized.

Every day of that week had pretty much the same scenario. I would start out in the morning with a very leisurely breakfast. Students from Saint Olaf College happened to be staying at this same hotel, and I eavesdropped on their conversations to see what phrases they were using. Then I would go to my room and work for about three or four hours. Then I would get to something I wouldn't know how to handle, and I wouldn't have any idea what to do next. So I would go for a walk around Oslo for about two hours. Maybe I'd go to the library, but usually I'd just walk around watching people. Then the solution to the problem would present itself. I would go back to the hotel, and after two more hours of work, I would get over the hump, and I would magically be able to move a little further. Then I would have a nice relaxing dinner, watch Norwegian television for about an hour, go back to the room, write some more, and put out the light.

The reason I think I had this muse was that the book would seem to write itself and fall into place. Things seemed to work out too nicely. I am, of course, very biased. But after I turned out the light, the next page would flash into my head, and I would have to get up quickly and write it down. The thoughts would come so fast that I would only have time to write the first letter of every word. Then I could turn out the light and sleep like a log. The next morning I would have to figure out what the first sentences were from the first letters of all the words. Every day the same pattern repeated.

The day I finished was the happiest day of my life. Oslo was so beautiful; there was a hoarfrost on all of the trees, more than an inch thick. I walked around in the gardens of the king's palace after having been to a movie with my wife. The frost-encrusted trees by the palace were magnificent. The midnight sky was a perfect, deep blue. I spent an hour gaping upwards in the park, marvelling at the patterns of trees against the sky, and then went back to the hotel. That was one of the greatest times I can remember. I knew that I was just one or two pages away from the end of the book. Then I finished the final chapter, except for a few unimportant mathematical details that I knew I could work out, and I relaxed into sleep.

The writing of *Surreal Numbers* had taken six days; so on the seventh day I rested. In fact, I still had the seventh day to tidy up the last page, which I did. Then I wrote "The End."

I couldn't write a word after that. I tried to compose a letter to Phyllis, my secretary at Stanford, telling her how I wanted this book to be typed. I would get into the middle of a sentence, and I could not figure out what verb to use. Suddenly, I couldn't even put simple things onto the page. I had just gone through a week where everything was sort of flowing out, and all of a sudden it was gone! That's why I love this book. It was a part of me that had to be expressed. I wish everyone could have a chance like this—some inspiration that could touch them.

Photo by Tom Black

When not playing the organ, computer scientist Knuth occasionally may be found playing in the backyard.

TYCMJ: *At the conclusion of "Surreal Numbers" there is a "dear teacher" letter, in which you suggest that students using the book for a course should do a project and write it up. In your last paragraph, you say that two of the major problems in teaching mathematics are a lack of experience in writing and also a lack of experience with creative thinking. Do you feel the same way today, several years later?*

Knuth: Oh, absolutely. I try to do that with our graduate students in computer science. We try to minimize the competition. We encourage working together on problem-solving, and discussing problems with each other. Creativity seems to be

The Roots of METAFONT

"Mathematics books and journals do not look as beautiful as they used to." With those words, Don Knuth introduced his 1979 article, "Mathematical Typography," to readers of the *Bulletin of the American Mathematical Society*. His statement clearly reveals an aesthetic concern about the physical appearance of mathematics.

This concern has resulted in his inventing TEX and METAFONT. TEX is described by C. Gordon Bell, Vice President of Engineering, Digital Equipment Corporation as follows:

"Don Knuth's Tau Epsilon Chi (TEX) is potentially the most significant invention in typesetting in this century. It introduces a standard language for computer typography and in terms of importance could rank near the introduction of the Gutenberg Press."

METAFONT is a system that makes use of classical mathematics to design alphabets. Knuth's aesthetic concern is clear when he says: "Of course it is necessary that the mathematically-defined letters be beautiful according to traditional notions of aesthetics. Given a sequence of points in the plane, what is the most pleasing curve that connects them? This question leads to interesting mathematics, and one solution based on a novel family of spline curves has produced excellent fonts of type in the author's preliminary experiments. We may conclude that a mathematical approach to the design of alphabets does not eliminate the artists who have been doing the job for so many years; on the contrary, it gives them an exciting new medium to work with."

Four years after he had started his research on METAFONT, Don's mother sent to him the alphabet book that he had enjoyed as a boy of 2 or 3. A page from that book is reproduced on the facing page. Note the *x*'s that Don placed by each serif in the K and L. The 7 inside the K is a count of the serifs of that letter. Clearly, his aesthetic and mathematical interests in alphabets go back to his early childhood.

K

kitten

L

lighthouse

encouraged until about the fifth grade, at least in the education of my children. There were a lot of creative things that they once were asked to do; but after fifth grade, schools seemed to say: "We haven't got time for that anymore, no time for you to discover anything for yourself. From now on you will have to absorb all this information the world has been developing."

That's wrong. We should teach people things with an emphasis on how they were discovered. I've always told my students to try to do the same when reading technical materials; that is, don't turn the page until you have thought a while about what's probably going to be on the next page, because then you will be able to read faster when you do turn the page. Before you see how to solve the problem, think about it. How would you solve it? Why do you want to solve the problem? All these questions should be asked before you read the solution. Nine times out of ten you won't solve it yourself, but you'll be ready to better appreciate the solutions and you'll learn a lot more about the process of developing mathematics as you go.

I think that's why I got stuck in Chapter One all of the time when reading textbooks in college. I always liked the idea of "why?" Why is it that way? How did anybody ever think of that from the very beginning? Everyone should continue asking these questions. It enhances your ability to absorb. You can reconstruct so much of mathematics from a small part when you know how the parts are put together. We teach students to derive things in geometry, but a lot of times the exercises test if they know the theorem, not the proof. To do well in mathematics, you should learn methods and not results. And you should learn how the methods were invented.

TYCMJ: *Occasionally today we have been getting close to the burgeoning area called artificial intelligence. Has your attention been attracted to that area at all?*

Knuth: I enjoy reading about it. Many of the algorithms in Volume Four are used in artificial intelligence to solve interesting problems. They turn out to be in one-to-one correspondence with things that electrical engineers use for other purposes, and again I enjoy bringing together two literatures that are talking about the same thing. The shortest path problem, for example, is something that arises in many different disguises. In artificial intelligence, we find algorithms for theorem proving and problem solving, expressed in a different language than other people will be using for the same kind of problems that they are encountering in electronics for wire routing, or something like that.

I'm not a specialist in artificial intelligence, but I think the most interesting thing about it is something that I can paraphrase from a book by Pamela McCorduck (*Machines Who Think*). She points out that the question used to be "Can computers think?" By now, however, everything that has been associated with thinking has been done by computers; and the only human accomplishment that computers *can't* do well are things that people do *without* thinking! This is so true. The things we do without thinking are the things that computers have never done or hardly done, like walking. To control a robot to walk like an ant walks, or to program a computer so that it will recognize a face when someone has grown a beard, are extremely difficult. Children can talk languages; computers can't even translate languages very well. All the things we do subconsciously are the things that artificial intelligence hasn't been able to do. That's the most striking thing about the subject now. The big mystery is what goes on when we are not thinking. How do ants do such complicated things with no leader telling them what to do? How do their small brains come to the decision of how they are going to communicate with each other and solve problems? That's way beyond what we know now. I am fascinated by that, but I never promised to write a book on it.

TYCMJ: *You're not terribly optimistic then?*

Knuth: I believe the study of artificial intelligence is really important in that we learn much more trying to find out how these things are done than we actually do by having a computer system doing them. Trying to automate something is a great scientific achievement. After you automate something, the important thing is what you learned in the process, not really that the computer can now do a complex job.

While you're trying to explain something to a computer, you have to understand it so well that it's even better than the understanding you get by teaching it to somebody else. The old saying is: "You don't learn something until you have taught it to someone else." Today's saying is: "You don't really know something until you have taught it to a computer." That's the secret of learning. The computer is really a good test of understanding. It doesn't allow you to wave your hands and say: "Now you use some common sense." You have got to understand it clearly—there's no room for wishy-washiness. That's why I believe computer science impinges on education. If students can teach something to a computer, then you know they have got it in their heads.

TYCMJ: *How long have you been working on TEX and METAFONT?*

Knuth: Spring of '77 is when I started, so it will be four or five years by the time I'm done. I thought it would be a one-year project.

TYCMJ: *Do you think it was time well spent?*

Knuth: Yes, I think the things I've learned are really exciting, and they are causing a lot of good waves in the printing world. I think it happens fairly often that a person from one field, say a mathematician, will stumble into another field. He'll have a different background than the people in the other field, and so he can contribute new insights. Sometimes such people will change fields and change their life's work. Sometimes they just make the contribution. For example, I have heard that Larry Shepp's daughter developed cancer of the brain. He got interested in that problem, and worked on a technique for locating cancers that's actually turned out to be an important breakthrough. You'll see this happen a lot of times for one reason or another: a mathematician will wander into some other area, and he'll see from what he knows that it is possible to apply ideas to that area right away, and that will help those people a lot.

In my own case, some of the things I learned about typesetting seem to be important enough that I've really gotten excited about them, and that is why it is taking me four or five years instead of one. If the ideas hadn't worked very well, I would have just kept them to myself and not bothered telling anybody anything about them; I would just have used them for my own purposes, and that would have been enough. But several ideas from mathematics and computer science now seem important to typography, so I want to do my best to refine them and explain them well. At the same time I am anxious to return to Volume Four before it becomes too big to write.