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A Joyful Passion for Proofs: The Pied Piper of Mathematics

The Man Who Loved Only Numbers: The Story of Paul Erdős and the Search for Mathematical Truth

By Paul Hoffman. New York: Hyperion, 1998. 302 pp. Cloth, $22.95. Paper, $12.95.

Ever since Plato recorded how Socrates elicited mathematical wisdom from Meno's untutored slave, philosophers and educators have struggled to discover the source of mathematical understanding. Like the search for the source of the Nile, this exploration moved swiftly from the open landscape of childhood arithmetic to an undulating wilderness of abstract mathematics. As the clues to the source of mathematics became more numerous, they also became more obscure. To interpret these confusing signs, the expedition recruited a

crew of experts that included philosophers, mathematics educators, cognitive psychologists, and research mathematicians; recently they have added neuro-scientists and evolutionary biologists. But as these experts explore different tributaries of mathematics in their search for the source of understanding, they have fallen to arguing among themselves about the significance of their find-ings. Indeed, in recent years these arguments have become so heated that the media now describe them as math wars.

The challenges facing the mathematics expedition are daunting. Clues range from children who learn mathematics with little apparent effort, often without instruction, to those who become so anxiety-ridden that they exhibit classic signs of phobia (Buxton, 1991); from calculating prodigies to the pathology of acalculia (Dehaene, 1997); and from genius scholars such as Newton and Gauss to isolated savants such as Ramanujan (Kanigel, 1991). Evidence from evolu-tion suggests at best a very primitive genetic capacity for arithmetic (Butter-worth, 1999), yet evidence from experience suggests stunning and dispropor-tionate variability in mathematical capability arising from similar environments (Changeux & Connes, 1995). Researchers exploring this confusing terrain welcome every new lead, however obscure, with the anticipation that it may help unlock the mystery of mathematical understanding.

Clues to the source of mathematics abound in Paul Hoffman's book about the strange man who, it is said, loved only numbers. This man, known to friends and acquaintances as Uncle Paul, was the kind, quirky, childlike, homeless mathematician Paul Erdős. An incessant world traveler, Erdős regularly dropped in on mathematicians, often without invitation or notice, to ask and answer mathematical questions in return for a few nights' room and board. "My brain is open" was his greeting; "another roof, another proof" was his promise.

Erdős's penetrating Socratic conversations led him to publish nearly 1,500 papers with nearly 500 different coauthors. No mathematician in history has ever worked so productively with so many different people, and only one—the prolific 18th-century Swiss genius Leonhard Euler—has produced as much mathematics. Like a chess master, Erdős sometimes carried on dialogs with several mathematicians simultaneously, each working on different problems. When he was not talking mathematics he was writing letters about mathemat-ics at a rate of more than 100 per month. Even in his 70s he often worked at mathematics 18 hours a day, publishing an average of one paper per week. Erdős was mathematics' consummate problem poser and problem solver.

But Erdős, who died in 1996 at age 83, was also a living caricature of the stereotype of a mathematician. Child prodigy? At age 3 Erdős taught himself to multiply three-digit numbers in his head, amusing his mother's friends by mentally calculating how many seconds they had lived. Nerd? Erdős was slight and gaunt (130 lb), with the "cadaverous look of a drug addict" (p. 8). He wore thick-rimmed glasses and typically dressed in old sandals and a rumpled gray suit. Mnemonics? Erdős once looked up six phone numbers, got distracted by conversation for half an hour, then dialed all six from memory. Obsessive? Erdős hated to be touched, washed his hands twice an hour, and dried them by shaking water on the floor rather than letting a towel touch his hands. Ec-centric? Erdős reveled in using a private language wherein *boss* meant woman

and *slave* meant man, Sam was the United States, Joe the USSR, and the Supreme Fascist (SF) was God. His common greeting, "my brain is open"—itself the title of another biography (Schechter, 1998)—meant that he was ready to do mathematics and he expected you to be ready as well, even at 5:00 a.m.

Addiction? Especially in his later years, Erdős lived on Benzedrine, Ritalin, and caffeine. A mathematician, he said, is "a machine for turning coffee into theorems." Helpless? Erdős didn't learn to tie his shoes until he was 11, hadn't buttered his own bread until he went to college, and never learned to boil water for tea. Throughout his life all his needs were taken care of by others—by his mother and, after her death, by his friends. Detachment? Erdős didn't see a movie or read a trade book in his last 50 years, nor did he show any interest in things such as food and sex, which are normally indispensable to human life. Freudian? Erdős never married, never had sexual relations, and abhorred pictures of naked women. In fact, he traveled with his mother for the last decades of her life. (Truth be told, he loved his mother more than he loved numbers.)

Yet as Hoffman richly documents, this uncommon man—one of the 20th century's authentic eccentrics—turned out to be not only the second most prolific mathematician of all time, but also a warm, generous, gregarious, and selfless human being. He cared deeply about people, especially children, whom he called epsilons (after the Greek letter mathematicians use to represent small quantities). Wherever he traveled, Erdős sought out mathematically talented youth and "poured problems" on them. To motivate potential problem solvers, Erdős regularly offered rewards for solutions to some of his more vexing problems and used the money he earned from lecture fees to fulfill these promises. All told, this self-sacrificing savant whose entire worldly possessions filled half of one battered suitcase awarded more than $15,000 for solutions to problems that he posed.

Erdős regularly sought to support anyone who appeared vulnerable. At one reception that included a mathematics student who had cerebral palsy, Erdős focused his attention on this student, helping him immeasurably with his work. At another party, upon learning that the host's blind father was alone in an upstairs room, Erdős spent the entire evening talking with the father. In 1945, he spent 2 weeks throwing mathematical problems at Stanislaw Ulam in a successful effort to help him recover mentally from an encephalitis-induced coma he contracted while working on the atomic bomb at Los Alamos. Similarly, but with less success, Erdős spent weeks trying to help the depression-prone logician Kurt Gödel regain his lost mathematical confidence.

The Man Who Loved Only Numbers is more memoir than biography, more collage than narrative. Filled with affectionate anecdotes about Erdős's life and habits, it offers an impressionistic rather than representational portrait. Author Paul Hoffman, publisher of the *Encyclopaedia Britannica,* began studying Erdős in 1986 when he wrote an award-winning profile of Erdős for *The Atlantic* (Hoffman, 1987). Hoffman interviewed many of Erdős's friends and colleagues, visited with Erdős on several occasions, and observed firsthand the impact Erdős had on those he touched, from young children to widows of childhood friends, from high school mathematics students to world-class research mathematicians. Hoffman himself describes the book as an oral history based on the recollec-

tions of Erdős, his collaborators, and their spouses. He has even created a Web page (http://www.paulerdos.com) to display many facts and artifacts of Erdős's remarkable life. (Many Web sites are devoted to aspects of Erdős's life and work; a reasonably complete list can be found at http://www.oakland.edu/~gross-man/erdosdeath.html.)

Hoffman's book is not only a memorial to Erdős but also a tribute to the community of mathematicians who safeguarded this maddening, remarkable man. Foremost among these is Ronald Graham, who served as Erdős's American host, handler, and executor. Graham and his wife, Fan Chung, both mathematicians, handled all of Erdős's finances, received and forwarded his mail, and even built a guest room in their home for Erdős. Hoffman offers an extensive profile of Graham, whose athletic vigor contrasts sharply with Erdős's frailty even as their mathematical work is inextricably intertwined. Erdős and Graham "were like an old married couple, happy as clams but bickering incessantly, following scripts they knew by heart though they were baffling to outsiders" (p. 20).

As this slim volume is not a conventional biography, neither is it a serious account of Erdős's mathematical accomplishments. Although it does convey in lay terms a good deal of insight into mathematics, for the most part this is not Erdős's mathematics. Major sections are devoted to mathematical digressions that appear, as if obligatory, in nearly every popularization of mathematics: the evolution of numbers leading to Cantor's continuum hypothesis, Andrew Wiles's solution of Fermat's last theorem, G. H. Hardy's discovery of Ramanujan, Appel and Haken's computer-assisted proof of the four-color conjecture, and Kurt Gödel's shattering discovery that mathematics is either incomplete or inconsistent. Although some readers may find these meanders enjoyable, many others will find them an unwelcome distraction from the ongoing adventures of Uncle Paul.

More significantly, however, these digressions mislead readers about the true nature of Erdős's contributions to mathematics. Erdős's work has almost no connection with the elliptic curves used to prove Fermat's theorem or the intricate logic of Gödel's proof. Indeed, Erdős's mathematics has little to do with most of the profound accomplishments of 20th-century mathematics. His special genius was in proposing and solving problems in elementary mathematics— the kind of problems that can be worked on productively by anyone who thinks clearly, even without special knowledge of advanced mathematical theories. That is why he was able to have so many collaborators, why he was able to help so many young students. Erdős's accomplishments manifest Plato's belief that the germs of mathematical understanding lie dormant in everyone's mind.

Not surprisingly, much of Erdős's character was shaped by circumstances of his early life. Born in Budapest on March 26, 1913, Erdős was the son of two high school mathematics teachers. While his mother was in the hospital giving birth to Paul, his two older sisters, ages 3 and 5, died of scarlet fever. "No mother could recover from such a loss," recounted a cousin. "She never did."

Fifty years before Erdős was born, Hoffman reports, the Hapsburgs emancipated Hungary's Jews and encouraged their assimilation into the upper classes of society. One result was an unprecedented outburst of mathematical and

scientific talent nurtured in Budapest in the early years of this century. In addition to Erdős, the Budapest diaspora includes mathematicians John Kemeny and John von Neumann and physicists Leo Szilard, Edward Teller, and Eugene Wigner. As a youth, Erdős was surrounded by people of mathematical talent and by people who appreciated such talent. As an adult, Erdős devoted his life to recreating this atmosphere everywhere he traveled.

Part of the magic of Erdős's mathematical life was his devotion to elementary methods. Here *elementary* does not mean simple. Rather, it means reliance only on basic results of algebra and geometry that a good student (like Meno's slave) might be expected to know or figure out. Although Erdős was a master of advanced techniques that used infinite series, imaginary numbers, and reasoning with limits, he specialized in reasoning that rested on more primitive and intuitive foundations.

At age 18, Hoffman recounts, Erdős discovered an elementary proof of a well-known result in number theory: that between every number and its double there is at least one prime. A few years later he made perhaps his most famous contribution to mathematics: the discovery (with Atlie Selberg) of an elementary proof of the prime number theorem, a formula describing the distribution of prime numbers whose only previously known proof involved techniques from complex analysis that extend well beyond the intellectual terrain of the theorem itself.

Prime numbers are notorious for their irregularity; no formula has ever been devised that unerringly predicts where they occur. However, the prime number theorem shows that despite this unpredictability, some order is present. Erdős focused much of his work on questions of this sort, finding order in contexts where patterns seem unlikely. He made major contributions to Ramsey theory, a branch of combinatorial mathematics devoted to proving that every large structure necessarily contains some degree of order. The classic Ramsey problem is to determine the smallest group that ensures either that at least three people will know each other or that at least three will be mutual strangers. It turns out that 6 is the magic (Ramsey) number; 5 won't do, and 7 is more than needed. To guarantee similar patterns involving four friends or mutual strangers takes a group of 18. But for five or more, all anyone knows is that the Ramsey number is more than 43 but less than 49. The Ramsey number for six is vastly more perplexing, apparently well beyond the range of either human or computational power.

Ramsey theory illustrates many characteristics of Erdős's mathematics. The problems can be understood by a child, but the solutions require ferocious concentration. Erdős's problems are purely mathematical (some might say artificial), without apparent practical value. Yet as with everything else in mathematics, practical-minded folks often find important uses for these results. But Erdős was not one of them. For Erdős, mathematics—especially numbers—offered a poetry of order that transcends applications. Preoccupied with death, Erdős often proclaimed mathematics as "the surest way to immortality."

As Hoffman rightly informs his readers, many of the world's most creative mathematicians have teetered on the brink of madness, and some have fallen in. Georg Cantor, who introduced transfinite sets to mathematics, became de-

lusional. Bertrand Russell, who unsuccessfully sought to create a logical foun-
dation for mathematics, contemplated suicide. Kurt Gödel, who proved that not
all true mathematical statements can be proven, grew paranoid. Princeton
mathematician John Nash, who received the 1994 Nobel prize in economics
for a dissertation written nearly half a century earlier, spent most of his adult
life struggling with schizophrenia—as, apparently, did Unabomber Ted Kaczyn-
ski, who received a Ph.D. in mathematics from Michigan and taught mathemat-
ics at Berkeley. The world of pure mathematics seems to attract psychological-
ly fragile people who single-mindedly pursue ultimate beauty by seeking elegant
solutions to potentially unsolvable problems.

   Hoffman's engaging narrative suggests that Erdős also was a bit mad, but his
was a joyful madness. His entire life was a quest for proofs from what he called
"The Book," an imagined record of the most elegant proofs of all mathemati-
cal theorems. But instead of consuming himself with a solitary focus on find-
ing his own proofs, Erdős enlisted an army of eager volunteers to search the
wilderness for clues and help find proofs worthy of The Book.

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The Writer Illusion: Consciousness Is More Useful
Than You Think

The User Illusion: Cutting Consciousness Down to Size

By Tor Nørretranders. Translated by Jonathan Syndenham. New York: Viking,
1998. 467 pp. Paper, $29.95.

One of the deepest problems facing science today is that of consciousness: What
is its nature and how is it created by the activity of the brain? It has been ac-
cepted by most workers on these problems that a precise definition of con-