Core Curriculum in Context

History, Goals, Models, Challenges

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Text of the opening plenary address at a May 1994 Symposium at the United States Military Academy at West Point, N.Y. The subject of the Symposium was the core curriculum in mathematics for the first two years of college. This paper appears in the symposium proceedings "Confronting the Core Curriculum," John A. Dossey, editor, Mathematical Association of America, 1998 (MAANotes No. 45), pp. 3-13.

As the designated historian for this meeting, I chose to begin forty years ago at the founding of the MAA Committee on the Undergraduate Program in Mathematics (CUPM). One could begin earlier, for as you will see from the record, the issues we have come here to discuss are not new--neither now, nor in 1953 when CUPM was founded. Questions about integrating the mathematics curriculum--or any curriculum--are as perennial as any debate in the history of education.

CUPM was established in 1953 by the Mathematical Association of America "to modernize and upgrade" the mathematics curriculum and to halt "the pessimistic retreat to remedial mathematics." (Actually, at the time of its founding, the committee was known merely as the Committee on the Undergraduate Program, or CUP.) In 1953, the total college enrollment in mathematics courses was 800,000; bachelor's degrees in mathematics numbered 4,000; Ph.D's, 200. During the intervening four decades, there has been a four-fold increase in these numbers--to three million mathematics students, 16,000 majors, 1000 Ph.D's. Most of that growth occurred during the first half of this period in the ramp-up of science and engineering during the post-Sputnik panic.

You'll be pleased to learn that CUPM's first project was a proposal for a course whose motivation and character were quite similar to the themes on our agenda for this weekend. Called <u>Universal Mathematics</u>, this course was intended as an integrated introduction to higher mathematics that provided in the first semester continuous mathematics, and in the second, discrete mathematics:

- *First Semester*: Analysis, college algebra, introduction to calculus
- *Second Semester*: Mathematics of sets and elementary discrete mathematics

However, despite CUPM's urging, nothing much changed. Like similar proposals in more recent years, CUPM's new course was too radical. It emerged at a time when the typical curriculum at even the best institutions reflected an entrenched pattern that was by then several decades old:

First Year:	Trigonometry, College Algebra, Analytic Geometry
Second Year:	Differential and Integral Calculus
Third Year:	Advanced Calculus
Fourth Year:	Differential Equations, Theory of Equations, ...

This early foray into curricular reform contains an important lesson for others who may have similar ideas: curriculum proposals from committees succeed primarily when they are a synthesis of evolving consensus, not the leading edge of radical reform.

To understand the context in which the early CUPM functioned, we must recognize important parallel events that helped shape the climate for the first two years of college mathematics in the 1950's and 1960's. Foremost was SMSG, a campaign led by mathematicians and mathematics educators to improve school mathematics. Then there were seminal university texts: Thomas in calculus, Van der Waerden and Birkhoff & Mac Lane in modern algebra. Other factors, particularly the emergence of computing and the expanding horizons of applications, forced the mathematics community to confront the reality of a new context for the undergraduate mathematics program. Thus began the tradition of curriculum reports that have been the landmark not only of CUPM, but also of the discipline of mathematics. No other subject has as strong a tradition of systematic nation-wide examination of undergraduate curriculum as has mathematics. Figure 1 gives a sample of titles to illustrate the steady flow of reports that began in the early years of CUPM.

Four Decades of Curriculum Reports

(*Selected MAA Publications*)

1965	*A General Curriculum in Mathematics for Colleges*
1965	*Pre-Graduate Preparation of Research Mathematicians*
1971	*An Undergraduate Program in Computational Mathematics*
1972	*Commentary on "A General Curriculum in Mathematics for Colleges"*
1972	*Introductory Statistics without Calculus*
1972	*Undergraduate Mathematics Courses Involving Computing*
1981	*Recommendations for a General Mathematical Sciences Program*
1986	*Toward a Lean and Lively Calculus*
1987	*Calculus for a New Century*
1989	*Discrete Mathematics in the First Two Years*
1991	*A Call for Change: The Mathematical Preparation of Teachers of Mathematics*
1992	*Perspectives on Contemporary Statistics*

Figure 1

A General Curriculum for Mathematics

In 1965--the year I finished my Ph.D.--the National Academy of Sciences issued a report entitled The Mathematical Sciences that introduced a new umbrella designation to encompass the related disciplines of mathematics, statistics, operations research, and computer science. It also introduced the phrase "core mathematics" as an alternative to the controversial moniker "pure mathematics" to more accurately describe those parts of the mathematical sciences that have traditionally occupied the intellectual center of the discipline and of the curriculum.

That same year, CUPM issued its first comprehensive curriculum report: A General Curriculum in Mathematics for Colleges. In this slim yet influential volume, CUPM described a single curriculum that can serve equally well as a foundation for students with varieties of interests and for colleges with varieties of missions. The report began with a sober analysis of the challenges posed by the then-current curriculum:

> Many of us can well remember when a firm tradition decreed that college mathematics should consist of the sequence: College Algebra, Trigonometry, Analytic Geometry, Differential Calculus, Integral Calculus, Differential Equations, Theory of Equations, Advanced Calculus, ... That old structure, which comfortably regulated college mathematics, has fallen apart.

This CUPM report goes on to diagnose, in terms that sound uncannily contemporary, the causes of this collapse and the challenges it poses:

> The generally welcome revolution in school mathematics...has created a greater diversity of entering students than we ever experienced before...and it has only begun. The <u>spread</u> in the mathematical capability of entering students will become much greater still.
>
> There are now many more kinds of mathematical knowledge...brought about by the computer, the increasing mathematization of the biological, management, and social sciences, ... and by the modern emphasis on such subjects as probability, combinatorics, logic, ...
>
> Thus there is a multiple output as well as a multiple input to the mathematics department's "black box."

It's specific curriculum recommendations are summarized in Figure 2.

A General Curriculum in Mathematics for Colleges

(A 1965 Report of CUPM)

Lower Division Courses:

0. *Elementary Functions*. Polynomials, rational, algebraic, exponential, and trigonometric functions.
1. *Introductory Calculus*. Differential and integral calculus of elementary functions with associated analytic geometry.
2. *Mathematical Analysis I*. Techniques of one and several variable calculus; limits, series.

2P. *Probability*. Sample spaces, random variables, limit theorems, statistical inference.

3. *Linear Algebra*. Linear equations, vectors spaces, linear mappings, matrices, quadratic forms, eigenvalues, geometric applications.
4. *Mathematical Analysis II*. Multivariable calculus, linear differential equations.

Upper Division Courses:

5. Advanced Multivariable Calculus.
6. Algebraic Structures.
7. Statistical Theory and Inference.

7'. Probability and Stochastic Processes.

8. Numerical Analysis.
9. Classical Geometry.

9'. Differential Geometry.

10. Applied Mathematics.
11. Real Variable I.
12. Real Variable II.
13. Complex Analysis.

Figure 2

Among its many specific recommendations, the 1965 CUPM report

- Begins with elementary calculus based on prior study of elementary functions.
- Introduces multivariable calculus in the first year.
- Recommends linear algebra at the beginning of the sophomore year.
- Advocates introducing all students to "the type of algorithmic approach that enables a problem to be handled by a machine."
- Provides for multiple entry points, and multiple exits. Offers "a more suitable compromise between the whole of calculus and no calculus than does the conventional course structure."
- Economizes offerings to enable the entire major to be taught by a four person department.
- Starts intuitively, with increasing rigor. "Start where the student really is, and proceed to where he [or she] should be."
- Includes options for one-year mathematics requirements: Math 0-1, or 1-2P.

Reflections and Commentary

By 1972, CUPM reflected on what it had tried to accomplish in this first attempt at standardization of the undergraduate mathematics major. It did this by publishing a "Commentary" on the 1965 report in which it raised explicitly the question of formally shifting the focus of departments from "mathematics" to "the mathematical sciences": "When thinking about undergraduate education, is it not now more appropriate to speak of *the mathematical sciences* in a broad sense rather than simply of *mathematics* in the traditional sense?"

The 1972 *Commentary* reflected on the criticisms that had come to the Committee in the intervening years:

- The pace of the course outlines was too fast.
- The syllabi leave no room for applications.
- Departments also have other substantial commitments.
- Mathematics is broadening beyond its classical boundaries.
- Some traditional core courses are less relevant than newer applied courses.
- The lower-division service responsibilities of a mathematics department are not well met by a single calculus-based track.
- The increasingly diverse mathematical preparation of entering students requires multiple points of entry prior to calculus.

The 1972 *Commentary* concludes with an observation that virtually recants the title of the1965 CUPM report: "Thus...it is no longer clear that there should be a single general curriculum in mathematics."

Commentary contains a number of explicit recommendations that extend or revise what CUPM had proposed in the 1965 General Curriculum report. These include:

- An explicit retreat to the tradition of a basic two-semester calculus course during the first year of college mathematics. "CUPM does not wish to suggest any alternative for the first year of calculus."

- A recommendation that each exit should be logical and coherent. Math 1 should be a self-contained introduction to one-variable calculus, including the Fundamental Theorem of Calculus; Math 1-2 should be a self-contained introduction to the ideas of calculus of both one and several variables, "including the first ideas of differential equations."
- A reaffirmation of the placement of Linear Algebra (Math 3) in third semester. Argues for making Math 4 build more explicitly on the concepts from Math 3.
- A suggestion that Math 5--the fourth term of traditional calculus (vector calculus and Fourier methods)--is no longer necessary in the core for all mathematics majors.
- An urging that Introductory Modern Algebra (Math 6M) be part of the core for all mathematics students, and that colleges offer a sequel (Math 6L) that provides more advanced linear algebra.

Mathematics vs. The Mathematical Sciences

It is clear from both the 1965 and 1972 reports that CUPM's general philosophy for lower division mathematics was unambiguously designed to broaden the scope of mathematical exposure of students during their first two years in college: not only calculus, but also probability and linear algebra were recommended for these first two years. The move towards "the mathematical sciences" provided a philosophical context in which these recommendations were embedded.

However, this consensus of movement from mathematics as traditionally defined to the mathematical sciences broadly conceived was not the only force influencing undergraduate mathematics. Other CUPM panels in this same era produced a series of recommendations for students intending to pursue advanced study of mathematics that were rooted more in the spirit of Bourbaki, in which the central core of mathematics was a beautiful intellectual synthesis of analysis, geometry, and algebra.

Pre-Graduate Preparation of Research Mathematicians

(A Sample of Topics from a 1965 CUPM Report)

First year:	Second year:
Tangents, derivatives, parametric curves	Characteristic equation and eigenvalues
Integral as positive linear functional	Cayley-Hamilton theorem; Gram-Schmidt process
Proof that cont. functions are integrable	Open, closed, compact sets
Pointwise and uniform convergence	Completeness of space of continuous functions
Monotone convergence theorem	Integral as uniformly cont. positive linear functional
Linear transformations	Implicit function theorem
Vectors in two and three dimensions	Differential p-forms
Rank and nullity theorem	Divergence, curl, Stokes' and Gauss' theorems

Figure 3

The CUPM Panel on Pre-Graduate Preparation of Research Mathematicians, writing in 1965, offered an ideal program (see Figure 3) which was, as they admitted, "somewhat unrealistic," but nonetheless "suitable for honors programs" and "as a goal for the regular curriculum." Theirs was an explicitly mathematical vision emphasizing from the very beginning the intrinsic unity of the subject. "Calculus should be presented so as to introduce and utilize significant notions of linear algebra and geometry in the construction of analytic tools for the study of transformations of one Euclidean space into another. ... Material should be arranged and presented in such a manner that students are ever mindful of mathematics as an interrelated whole rather than as a collection of isolated disciplines."

Computational Mathematics

In 1971, anticipating the monumental changes that computers were going to bring to the practice of mathematics, CUPM issued a special set of recommendations for an undergraduate program in computational mathematics that was "intended to be a departure from the traditional undergraduate mathematics curriculum." This report, rooted in very different assumptions than those of the pre-graduate preparation report, recognized the need for "innovative undergraduate programs that provide for a wide range of options, for different opportunities for graduate study, and for a variety of future careers." The CUPM program in computational mathematics was intended to be one of "several equally valid options" for students in the mathematical sciences. By building on the 1965 "General Curriculum" this new program was specifically designed to "permit continuation in computational mathematics or in pure mathematics, with suitably selected senior courses."

Figure 4 shows the list of courses recommended for this program: twelve courses in mathematics (M1-M5), computing (C1-C3), and computational mathematics (CM1-CM4) to be taken during the first three years of undergraduate study. The mathematics courses are computationally slanted versions of the courses previously recommended by CUPM; the computer courses are versions of those recommended by the Association of Computing Machinery (ACM) in "Curriculum '68" (later revised in "Curriculum '78"). The four courses in computational mathematics represent a new hybrid intended to build substantive links between computing and mathematics.

An Undergraduate Program in Computational Mathematics

(1971 Recommendations of CUPM)

M1.	Introductory Calculus.	CM1.	Computational Models & Problem Solving.
M2.	Mathematical Analysis I.	CM2.	Introduction to Numerical Computation.
M3.	Linear Algebra.	CM3.	Combinatorial Computing.
M4.	Mathematical Analysis II.	CM4.	Diff. Equations and Numerical Methods.
M5.	Advanced Multivariable Calculus.		

C1.	Introduction to Computing.	<u>Year 1</u>: M1, M2, C1, CM1
C2.	Computer Organization and Programming.	<u>Year 2</u>: M3, M4, C2, CM2
C3.	Programming Languages and Data Structures.	<u>Year 3</u>: M5, C3, CM3, CM4

Figure 4

Statistics and Discrete Mathematics

Continuing its campaign to move mathematics toward the mathematical sciences, in 1972 CUPM published recommendations for "Introductory Statistics without Calculus." The purpose of such a course was primarily to introduce the ideas of variability and uncertainty, and--contrary to tradition--only secondarily to introduce standard formulas, terms, and techniques. This first course in statistics, CUPM argued, should emphasize inferential concepts and data analysis, not mathematical elements.

CUPM recommended several types of courses to achieve this objective. Each would employ real data sets and build on the computational power of computers for simulation, calculation, packages, and interactive learning. These recommendations emphasize hands-on methods, including demonstration experiments (e.g., tossing a coin 100 times) that illustrate predictable patterns; open experiments (e.g., tossing thumbtacks to see how they land) that illustrate empirical phenomena whose patterns cannot readily be predicted *a priori*; and simulations (e.g., of queues and servers) of patterns that can best be observed through computer methods.

The main-stream recommendation in this report describes a fairly traditional course, but CUPM added three rather different alternatives to stress the point that it was statistical thinking, not just a collection of formulas, that was the goal of this recommendation:

S1. *Elementary Statistics*. Statistical description; probability, random variables; probability distributions; sampling distributions; inferences about population means.

S2. *Decision Theory*. Bayes' strategies; significance levels; confidence intervals.

S3. *Nonparametric Statistics*. Chi-square; contingency tables; correlation; robustness.

S4. *Case Studies*. Modeling; computer simulation; open-ended, meaningful problems.

The message about the nature of introductory statistics seems to have a hard time being heard, or implemented. Twenty years after this 1972 report, a recent MAA volume entitled Perspectives on Contemporary Statistics (1992) asserts that "a wide gap separates statistics teaching from statistical practice." It recommends yet again, echoing the 1972 report, that instruction in statistics should emphasize data: analyzing data, producing data, and inference from data.

Also in 1972, in response to the growing importance of computing, CUPM published recommendations intended to nudge the content of mathematics courses in the direction of computational procedures by stressing algorithms, approximations, model building, and problem-solving processes. Four course outlines (MC0-MC3) offer computer-oriented version of M0-M3. One new course, Discrete Mathematics (DM), is introduced both to supplement the standard curriculum and to "serve well as a first mathematics course for students from many disciplines."

DM *Discrete Mathematics*. Set theory, permutations and combinations, pigeonhole principle, generating functions, difference equations, relations, graphs, circuits, paths, Eulerian and Hamiltonian paths, network flow problems.

So by 1973, the mid-point of CUPM's history, we find not a "7 into 4" problem but what amounts to a "20 into 4" problem. Between 1965 and 1973, CUPM had proposed twenty different introductory courses in mathematics, computing, and statistics, each with legitimate claims for being part of the mathematical repertoire of any serious student intending to study a mathematics-intensive field (see Figure 5).

CUPM Recommendations as of 1973

M-1.	Trigonometry and Algebra.	CM1.	Computational Models and Problem Solving.
M0.	Elementary Functions.	CM2.	Introduction to Numerical Computation.
M1.	Introductory Calculus.	CM3.	Combinatorial Computing.
M2.	Mathematical Analysis I.	CM4.	Differential Equations & Numerical Methods.
M2P.	Probability.		
M-DM.	Discrete Mathematics.		
M3.	Linear Algebra.		
M4.	Mathematical Analysis II.		
M5.	Advanced Multivariable Calculus.		
C1.	Introduction to Computing.	S1.	Elementary Statistics.
C2.	Computer Organization and Programming.	S2.	Decision Theory.
C3.	Programming Languages and Data Structures.	S3	Nonparametric statistics.
		S4.	Case Studies.

Figure 5

A General Mathematical Sciences Program

In 1981 CUPM issued a new report entitled Recommendations for a General Mathematical Sciences Program that set firmly in place the notion that mathematical sciences rather than mathematics is the proper subject of the undergraduate program. “CUPM now believes that the undergraduate major offered by a mathematics department at most American colleges and universities should be called a Mathematical Sciences major.”

In this report, CUPM argued that first courses should appeal to “as broad an audience as is academically reasonable.” “The mathematical science curriculum should be designed around the abilities and needs of the average student, with supplementary work to attract and challenge talented students.” To broaden the appeal of mathematics--at a time when the number of majors had dropped by over 50% from its historic high--CUPM argued that computer science, applied probability, and statistics should be “an integral part of the first two years of college mathematics.”

As part of this report, CUPM recommended three new courses that should fit into the first two or three years of the undergraduate curriculum:

> *Discrete Structures*. Combinatorial reasoning (graph theory, combinatorics) taught at the level of introductory calculus.

> *Applied Algebra*. Sets, partial orders, Boolean algebra, finite state machines, formal languages, semigroups, modular arithmetic, automata, enumeration theory, lattices. (Adapted from ACM's "Curriculum '68" and ACM's "Curriculum '78.")
>
> *Statistical Methods*. A post-calculus course emphasizing data (organization and description), probability (random variables, distributions, Law of Large Numbers, Central Limit Theorem), and statistical inference (significance tests, point estimation, confidence intervals, linear regression).

The most contentious issue in this 1981 report concerned not the first two years of the undergraduate program, but the last two. Recognizing the message being sent by students who had "voted with their feet," CUPM dropped the historical requirement of year-long courses in both algebra and real analysis as the upper division core of the mathematics major. They recommended, instead, at least one rigorous two-course sequence at the advanced level--which might be, for instance, in applied mathematics or in probability and statistics. This recommendation recapitulated at the advanced level the contrast revealed fifteen years earlier by the two very different CUPM proposals for the content of the first two years of college mathematics.

Discrete Mathematics

The 1980's witnessed a flurry of curricular exploration related to discrete mathematics, whose importance grew in proportion to the increasing demand for computer science. Discrete mathematics was to be the language of the information age, as calculus had been the language of the age of (Newtonian) science. The key curricular issue for mathematicians was whether it was possible to design a year-long course in discrete mathematics that could hold its own in head-to-head competition with calculus as a legitimate entry point for the study of college mathematics. While most institutions experimented with separate courses, a few tried to devise an integrated approach approximately in the spirit of CUPM's stillborn Universal Mathematics of the mid-50's.

At least three volumes published during the 1980's record the recommendations and experiences of individuals who led these experiments. A special committee of the M.A.A. concluded a study of this movement with the clear recommendation that "discrete mathematics should be part of the first two years of the standard mathematics curriculum at all colleges and universities, and should be taught at the intellectual level of calculus." They provided a rather standard course description:

> *Discrete Mathematics*. Algorithms, graph theory; combinatorics, induction, recurrence relations, difference equations, logic, introductory set theory.

and outlined two options for implementation:

- Two one-year sequences in discrete mathematics and in (streamlined) calculus;
- A two-year integrated course in discrete and continuous mathematics (calculus).

Students and Courses

Curriculum recommendations that concentrate on calculus, statistics, discrete mathematics, linear algebra, and differential equations--the classic "7 into 4" problem--ignore the most important reality: students. It turns out, if one looks at the data (Figure 6), that most students take other courses--primarily those that are part of the traditional high school curriculum. Only one in three beginning college mathematics enrollments is in any of the seven courses we will be discussing. So in planning these core courses, we should not fall into the trap of assuming either that all able students are in one of these courses, or that all mathematics courses suitable for beginning students are included among the seven on our list. The reality of matching students and courses is far more complex: mathematics students do not simply enroll in courses, but are created by courses.

The Enrollment Facts of Life

(1990-91 CBMS Survey)

	Two-Yr	Four-Yr	Total	
Remedial	724	260	984	
Alg & Trig	245	360	605	
Finite/Busi/LA	90	173	263	
Elem Sch teach	9	62	71	
Technical Math	18		18	
Precalculus Level			**1941**	
Calculus	124	545	669	
Diff Eq	4	40	44	
Discrete	1	17	18	
Linear Algebra	3	42	45	
Calculus-Level			**776**	
Elem Stat	47	87	134	
Elem Probab	7	32	39	
Statistics & Probability			**173**	
Comp & Soc	10	69	79	
Packages	21	73	94	
CS1	32	80	112	
CS2	12	23	35	
Other Elem CS	23	86	109	
Computer Science			**429**	
Total Elementary Courses				**3319**

Figure 6

Goals and Objectives

Because there is so much overlap in both content and context between the mathematics taught in high school and the mathematical sciences covered during the first two years of college, it is useful to examine the goals of the curriculum from both school and college perspectives. The best known statement of goals, of course, can be found in the NCTM Standards for School

Mathematics. This influential document identifies five broad goals for mathematics education that define what NCTM terms "mathematical power":

- To reason mathematically
- To communicate mathematically
- To solve problems
- To value mathematics
- To develop confidence

In contrast, the following goals from a draft of standards being developed by the American Mathematical Association of Two Year Colleges (AMATYC) focus on empowerment of students--what one might term "student power":

- Empowerment: Increase participation in mathematics-based careers by students heretofore under-represented in those fields.
- Confidence: Provide rich, deep experiences that encourage independent exploration, build tenacity, and reinforce confidence in each student's ability to use mathematics effectively.
- Connections: Present mathematics and science as a developing human activity that is richly connected with other disciplines and areas of life.
- Citizenship: Illustrate the power of mathematical and scientific thinking as a foundation for independent life-long learning.

Turning to the core curriculum itself, many MAA reports identify at least four distinct missions that must be served by introductory college mathematics:

- To ensure numeracy for all college graduates.
- To provide students with mathematical skills for further study and work.
- To prepare prospective teachers to implement a standards-based curriculum.
- To attract able students to major in the mathematical sciences.

Strategies for Success in Undergraduate Mathematics

(from diverse NCTM, AMATYC, and MAA reports)

- Teach in ways that engage students and encourage learning.
- Provide for the mathematical needs of all students.
- Engage technology in substantive support of mathematical practice.
- Blur early distinctions between majors and non-majors.
- Build smoothly on the standards-based core school curriculum.
- Motivate theory with applications.
- Provide extensive opportunities for students to read, write, listen, and speak.
- Help students learn how to learn mathematics.
- Recognize that content and pedagogy are inseparable.
- Use assessment to reinforce the goals of instruction.

Figure 7

All the standards recommendations--from NCTM, from AMATYC, from MAA--agree on certain strategies that are essential to achieve any measure of success in college mathematics. Unfortunately, these strategies (see Figure 7) are honored more in rhetoric than in reality. Yet the considerable investment of recent years in reform of mathematics education has taught us many lessons about what works and what doesn't. As we embark on discussions about compressing seven courses into four, or reinventing the goals of introductory college mathematics, we would do well to heed some of these lessons from practice and research, from reform and accomplishment (see Figure 8).

Lessons Learned about What Works in Undergraduate Mathematics

From Practice:

- Learning takes place when
  - -- The student is enmeshed in community;
  - -- The subject is embedded in context;
  - -- The instruction is infused with inquiry.

From Research:

- Learning is construction of knowledge, and construction of knowledge is construction of motivation.
- Mathematical knowledge is not merely remembered, but is privately constructed, becoming unique to the individual.

From Calculus Reform:

- Writing to learn is as important as learning to write.
- Technology is the most powerful transforming agent.
- Mathematical models are tools for understanding.
- Group work encourages cooperative approaches.
- Multiple representations aid student understanding of mathematics.

From Successful Programs:

- View teaching as a collective responsibility of the entire department.
- Make teaching a public activity, supported by regular discussion and seminars.
- Form natural communities on educational issues as a context for peer review.

Figure 8

Models

It is not for me to talk about specific models, but for you all: that is what this conference is all about. I would, however, like to set the context for your thinking about models by stressing the diversity of higher education--of institutions, of programs, of student goals, and of student preparation. Since students typically enroll in mathematics as part of a defined program of study, the context of these programs is of crucial importance to the nature and goals of introductory college mathematics.

Diversity of institutions and programs is one of the distinctive strengths of the American system of higher education. Students study mathematics in over 3000 post-secondary institutions of widely differing missions and purposes:

- Research Universities
- Comprehensive Universities
- Liberal Arts Colleges
- Community Colleges
- Vocational Institutes
- Employee Training Programs

Their programs of study are equally diverse, with widely different mathematical expectations depending on the particular purpose:

- Liberal Arts
- Humanities
- Pre-Law and Pre-Med
- Science and Engineering
- Business
- Education
- Vocational
- Computer Science

Mathematical Preparation of Entering College Students

A. Course Completion of High School Graduates
B. Course Completion of College Students
C. Achievement Inferred from College Enrollments
D. Cumulative Course Completion of High School Graduates
E. Cumulative Course Completion of College Students
F. Cumulative Achievement Inferred from College Enrollments

A	B	C	Level	D	E	F
14%	0%	12%	Arithmetic	100%	100%	100%
15%	10%	22%	Algebra I	86%	100%	88%
20%	25%	18%	Geometry	71%	90%	66%
25%	30%	20%	Algebra II	51%	65%	48%
18%	25%	21%	Precalculus	26%	35%	28%
8%	10%	7%	Calculus I	8%	10%	7%

Figure 9

Figure 9 provides a profile of student preparation as they begin their study of college mathematics. The first three columns display percentages of students prepared at different mathematical levels at three different stages of the school-college transition: upon graduation from high school, upon entrance to college, and upon enrollment in mathematics courses. One can see first that students leave high school with a considerable range of preparation in mathematics, that those who enter college are better prepared mathematically than the typical high school graduate, but that the range of demonstrated competence of college students based on college course enrollments is much weaker than the recorded preparation based on high school courses completed.

Challenge I: An Integrated Course

This conference is intended to address two basic questions. The first is about the content of the core: "How to integrate the critical content of calculus I, II, III, linear algebra, differential equations, probability and statistics, and discrete mathematics into a four-course sequence."

As we have seen, this question has a long history, My recital began nearly forty years ago with the CUPM proposal for a course in Universal Mathematics, but the vision of an integrated introductory course has deep roots in nineteenth century European curricula. In this era we have several contemporary experiments motivated by the same vision: The College Board's "Pacesetter" program provides an integrated modeling-based pre-calculus course, while COMAP's "The Foundation" provides a similar modeling-based integrated approach to first year college mathematics.

The challenge of these courses is to become mainstream. In contrast to many other college subjects (e.g., physics, chemistry, economics), mathematics has no tradition of beginning the undergraduate curriculum with a "Principles of Mathematics" course. This idiosyncrasy may be related to another distinctive characteristic of mathematics--that it is one of only two university subjects (the other being English) that builds in essential ways on a full K-12 curriculum. Students' mathematics education is in full swing by the time they enter college--which makes it difficult to develop an effective introductory course based on the ideal of "one-size-fits all."

The other more obvious reason that integrated introductions to mathematics have never succeeded is the controlling influence of engineering and its mathematical prerequisites. The tradition of beginning with a particular course--calculus--is largely due to the needs of engineering students in the major universities. But now that students take college mathematics for many other reasons as well, it is appropriate to take up the challenge of alternative courses. It is, however, more likely that evolution will dictate a bush-like structure to the curriculum (e.g., choices of entry points including calculus, discrete mathematics, statistics, and computing) rather than a new tree whose trunk is a planned, integrated course that serves as a substitute for calculus.

Challenge II: A Compelling Course

The second question posed by this conference is, in my judgment, more apt and more critical: "What is the role played by a set of fundamental core courses in launching the study of mathematics for students majoring in the mathematical sciences or in mathematically-dependent fields?" This question is interesting because it focuses on the student, not the subject. It turns our attention away from the aesthetics of curriculum design to the practical reality of how the curriculum can attract (or repel) prospective mathematics students. A moment's thought produces a host of difficult yet important questions:

- How do introductory courses influence student decisions about majoring in mathematics-intensive fields?

  (To succeed, students need to be welcomed into a community in which they can grow in confidence while supported by a safety net of friends and faculty who help them overcome mistakes and insecurity. In what ways can the curriculum, and its implementation, accomplish this important task of mathematical acculturation?)

- How can departments provide multiple entry points to diverse curricular paths that lead to productive careers?

  (Clearly, courses that lead to curricular dead-ends should not be offered. But how, for instance, can a student who begins with statistics and gets "hooked" move on to complete a major in the mathematical sciences in a natural progression that builds on the foundation of statistics rather than of calculus?)

- How can course sequences be planned to serve well those, often the majority, for whom the course they are currently in will be their final mathematics course?

  (Ideally, each course must address the broad goals of mathematics education, and must leave students with a positive attitude about mathematics. Can this be done without compromising the "coverage" requirements of each course?)

- How can first year courses be organized to benefit equally students from standards-based school programs as well as those from traditional programs?

  (Students who enter college in coming years may come prepared for a robust, hands-on, modeling approach to mathematics; others will come with traditional expectations of a paper-and-pencil problem-oriented course; still others will come with chaotic mixtures of skills, accomplishments, and expectations.)

- How can departments ensure that different tracks all meet similar broad goals and ensure flexible future transitions for students who change career goals?

  (One strategy is to reduce prerequisites to a minimum and introduce specific background on an as-needed basis. That way students will be encouraged to work for broad objectives rather than meeting narrow and oftentimes somewhat arbitrary prerequisites.)

Criteria

Finally, I leave you with three criteria by which to judge proposals for a core program. You may think these are mutually contradictory. But I claim that they are logically necessary if we are to succeed with undergraduate mathematics.

- Core courses should serve equally well all students in every course.
- Core courses should attract students to continue the study of mathematics.
- Core courses should launch new students into mathematics-intensive fields.