

How School Mathematics Can Prepare Students For Work, Not Just for College

Curricula that focus on real-life applications of concrete mathematics can meet both academic and workplace expectations

BY SUSAN FORMAN AND LYNN ARTHUR STEEN

A carpenter's plan for a built-in kitchen shelf unit calls for 1" x 8" oak boards of four different lengths, ranging from 2'10" to 6'6". The local lumber yard sells oak boards in lengths of 6, 8, 10, 12, and 16 feet, but the shorter lengths are more costly per board foot. The carpenter must decide what combination of shelf lengths and purchased boards will be most economical.

A financial assistant in a major HMO must make projections about the effects of increasing co-payments while broadening the scope of coverage, all without changing premiums. Using a standard spreadsheet template that covers all the HMO group policies, he locates the cells where projection calculations are made. He must study these cells to understand how their formulas now work, then modify them to reflect the proposed changes, and finally run several test cases to be sure his changes do what was intended.

A truck driver for an appliance company must plan the day's schedule and load a van whose cargo space measures 9'3" x 6'6" x 7'. The day's deliveries include four refrigerators, each in a 34" x 34" x 68" carton; three stoves, two in 32" x 28" x 50" cartons and one in a 32" x 28" x 74" carton; and a freezer in an 80" x 30" x 30" carton. The driver also has to pick up two television sets and a washing machine for repairs. Before setting out, he will use a map to plan his route and to notify each customer of the estimated pickup or delivery time.

These situations are examples of mathematics in context. Real-life problems are, typically, very concrete but not necessarily very straightforward. Generally they can be solved in many ways, and do not have unique "correct" answers. Several strategies may be "good enough," even if one may be technically a bit better than others.

Solving problems in actual work situations—unlike the short exercises found in most mathematics texts and classrooms—usually involves data with

realistic measurements expressed in common units. The technical skills required to solve such problems are fairly elementary: measurement, arithmetic, geometry, formulas, simple trigonometry. The problem-solving strategies, however, often require a sophistication that few students get from current school mathematics: planning and executing a multi-step strategy; considering tolerances and variability; anticipating and estimating relevant factors not immediately evident in the data; careful checking to assure accuracy.

Concrete Mathematics

Mathematics in the schools still suffers from public rejection of the "new math," which brings up images of set theory and spelling drills on the word *commutative*. Whereas "new math" tried to bring out the power and beauty of general, abstract mathematics, math at work is concrete. It is spreadsheets and perspective drawings, error analysis and combinatorics.

Critics worry that a curriculum focused on useful math will deny opportunity to talented students.

Concrete mathematics gives students a stronger basis for abstract thinking in later courses, a greater appreciation for the utility of mathematics, and a better understanding of the discipline. Its "what if" analysis provides opportunities for students to formulate and test their own hypotheses.

Unfortunately, concrete mathematics can easily be misinterpreted as merely the old "general math" in modern disguise. Courses in practical mathematics (called "consumer math," "general math," or "shop math") have always been held in low esteem. Too often such courses emphasize easy

"cookbook" solutions, attract poorly prepared and unmotivated students, are taught by teachers with little interest in mathematics, and provide narrow skills of little benefit beyond classroom exercises. Such courses are rightly disparaged as dead-ends. Instead of expanding students' horizons, they limit choices.

Concrete mathematics is not cookbook math. It is specific but not narrow, focused but not prescribed. It is found embedded in rich, authentic examples that stimulate students to think mathematically. Like personal anecdotes in politics and characters in literature, concrete cases are what one remembers. Concrete, practical mathematics works. The challenge for schools is to make what works respectable.

Mathematics for All

In an earlier time, when people rarely changed jobs and occupational skills were well-defined, school mathematics branched in the middle grades into two tracks—a one- or two-year "terminal" track for students not planning on college, and a full four-year curriculum for those who are. That time of simple, fixed career paths is long gone. The National Council of Teachers of Mathematics (NCTM) now recommends that the first three years of high school mathematics be designed for *all* students, whether or not they plan to attend college.

Although dictionaries define mathematics as the science of space and number, in fact it is more the science of patterns. Math provides a common framework for analyzing the mysteries of daily life. Exploring, estimating, classifying, and optimizing are useful for personal financial planning, for making business decisions, for interpreting public policy debates.

Mathematics enables us to represent relationships, and thus to make plans that can be trusted; to classify behavior, and thus to separate the predictable from the random; to model processes, and thus to anticipate the conse-

quences of our actions. As in earlier centuries when mathematics emerged as the language of science, so now a different mathematics has become the language of the technical work force.

The response of schools to new developments in mathematics should not be simply to add new topics to an old curriculum. We must instead focus on the forces for change in mathematics, not on the results of those changes. Instead of asking "What's new?" we should ask "What's newly useful?" Forty years ago, only engineers needed to know about digital electronics; now every technician must. Today's efforts to flatten management structures are changing many blue-collar jobs into "white-smock" jobs in which workers are expected to make decisions based on data continuously gathered from the work environment.

Watered-Down Curricula?

Emphasizing math for work gives us perspective on the view that broadly implemented standards will result in a watered-down curriculum. Those who express this fear argue that, in the name of "mathematics for all," talented students are being denied the opportunity to develop advanced skills of reasoning and symbol manipulation. They worry that if the entire curriculum is focused too narrowly on what is deemed useful for all students, then topics whose payoff is years away (like mathematical induction and the binomial theorem) will be neglected, and the better students will suffer.

But work skills require a sophistication and precision that can push even the best students to levels well beyond those found in most of today's classrooms. As they secure a broad foundation of examples and concrete mathematics, students will build their own connections between mathematics and the world in which they live and work. A grounding in specifics leads naturally to more abstract thinking. Workplace mathematics mirrors good pedagogy by moving from the specific to the general, from the concrete to the abstract.

Moreover, courses that put mathematics in context improve student motivation, and thus improve learning. Students will discover in their own experiences answers to the question "How am I going to use this?" Changing from a traditional symbol-intensive curriculum to one embedded in authentic work situations will help teachers to identify and encourage

students who reveal nontraditional aptitudes for math.

Knowing Versus Doing

The central intellectual issue in mathematics education is not pedagogy but content. We know what is required for effective teaching: active engagement with mathematics in context, supported by a sense of community that provides necessary meaning and motivation. We know much less about content. What is it that all students must learn? Do the particular topics learned really matter?

Some answers are easy: we want all students to learn to reason and calculate, to solve problems, and to communicate mathematically. Other questions are harder: do we want all students to be able to prove results, or to be able to solve quadratic equations?

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The key argument is about process versus product. Most people now recognize that doing mathematics (reasoning logically, solving problems) is more important than just knowing it (remembering formulas, memorizing algorithms). Yet this simple idea is radical. For teachers, it implies engaging students in the processes of mathematical thinking rather than in simply learning facts and formulas. This revolution in perspective is exactly what the NCTM hopes to accomplish through its standards: ensuring that students can *do* what formerly they were expected only to *know*.

The shift from product to process, from knowing to doing, requires that students engage mathematics as a whole, not just as a collection of separate topics. Although the development of mathematical expertise traditionally has been approached by decomposition into component skills, it is becoming clear that this "one rule at a time" approach does not work. "It is harder, not easier, to understand something broken down into all the precise little rules than to grasp it as a whole," writes mathematician William Thurston.

Thurston's advice for effective teaching derives from his experience, but it is the same advice we get from cognitive

science, educational research, and workplace practice: present mathematics in a way that is "more like the real situations where students will encounter it in their lives—with no guaranteed answer," he says. "It is better to keep interesting unanswered questions and unexplained examples in the air, whether or not students, teachers, or anybody is yet ready to answer them."

Shifting the Balance

With the publication of the NCTM standards came a philosophical and practical shift in the balance of leadership. No longer are colleges in the driver's seat in setting goals for math education. Now school teachers and mathematics educators are calling the shots. But the new standards, like what they are meant to replace, still respond to the siren call of traditional calculus-prep mathematics. High school students are still identified as "college-intending" or not; the core curriculum for all students is to be the same for eleven grades—the curriculum taken by those identified as college-bound.

The motives for this recommendation are laudable and the logic nearly inexorable: if all students are to have equal opportunity, all must receive the same education, an education previously reserved for the "elite." Yet three out of four students move into the work force along a path that does not depend on a four-year degree. The mathematics most students will study after high school will be very different from traditional college mathematics and may not involve calculus at all.

A plumber is not a failed engineer. Going to work should not be interpreted as a failure to go to college. Neither are practical courses necessarily lesser versions of academic courses. Both can provide challenge, excitement, and significant education; both can serve students' search for vocation with dignity and respect. In shifting educational priorities from *some* students to *all* students, the NCTM standards implicitly accept the challenge of educating prospective plumbers as well as future engineers.

Students' experiences in school should reflect the world of work, and a strong program that prepares students for work will also equip them for success in college. These are the standards by which schools should be judged, and should judge themselves: to provide a coherent program in which work is a natural extension of study, and

school is a natural foundation for work. Concrete mathematics will thrive in such a program, and so will students.

For Further Information

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