Algebra for All: Dumbing Down or Summing Up

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My task--to sum up the manifold discussions of the last three days--is necessarily inchoate. Each of us takes from this conference a uniquely personal perspective based on the particular working groups and discussions in which we participated. I do not pretend that my remarks will provide either a synopsis or an executive summary. They are, rather, a personal perspective on what I have heard these last few days. I'll try to follow the conference mandate: to focus on a few big ideas, especially by providing a policy perspective, emphasizing open questions, disputes, and omissions.

My notes from the discussions cover an enormous variety of topics--far too much for a one hour summary. So I adopted two principles of selectivity:

-- Omit ideas that have been widely discussed elsewhere during the last decade.
-- Omit issues that are not specific to algebra

One consequence, for instance, is that I have made no attempt to cover the workshop's wide-ranging and enormously helpful discussions about teacher education--not because it is less important, but because it is part of a larger discussion that has been going on for quite a number of years and because it is not, except in rare instances, uniquely related to the subject of algebra.

These two filters reduced my notes by at least two orders of magnitude. What emerged was not much about AFA ("Algebra for All"), much less about the proposed new acronym of AFAFA ("All for Algebra for All"). Most of our discussions seemed to be about everything *except* algebra: teaching, literacy, modelling, mathematics, standards, etc. The few drops of new ideas about *algebra* that these filters let through from the gush of words at this workshop suggest that a more apt acronym might be AA--not as some have suggested "Algebraists Anonymous," but more accurately, "Avoiding Algebra."

What is Algebra?

I'll begin with a brief review of various definitions of algebra, based primarily on the lectures by Victor Katz and Mike Artin:

-- Calculating by rules (*Early Islamic*)
-- Determining unknowns
-- Computations by signs and symbols (*15th-16th C.)*
-- Employing letters in reasoning about numbers
-- Working with '*x*' *(The typical student's definition)*
-- Study of arithmetic operations
-- Simple and general concepts, not cumbersome computations *(20th C.)*
-- Study of general operations and operators
-- The language of mathematics (and of commerce, ...)

Prior to Emmy Noether, algebra was what most students believe it still is today: calculating with letters to solve equations for unknowns. Only in the twentieth century did the more general notions arise that gave algebra its great power--the power to become, in Mike

Artin's words, the "language of mathematics." Of course, it is also the language of commerce and of technical discourse in many other areas of knowledge as well.

Before moving on I should remind you of a few of the lessons of history embedded in Victor Katz' very informative opening lecture:

- Historically, algebra was taught through artificial problems.
- Historically, algebra was taught to the few, not the many.
- Historically, algebra was taught not to find answers to useful problems, but to develop general problem solving skills.
- Historically, algebra of the twentieth century evolved from--hence requires mastery of--the algebra of previous centuries.

These lessons lead to a key observation, and a significant warning: "Algebra for all" taught through realistic problems with reduced emphasis on complex manipulation to provide skills needed for work violates all these lessons of history. Are we really ready for such a revolution?

Major Themes

The major theme of this conference is very simple: Algebra is broken but nonetheless essential. Algebra I--the current manifestation of "Algebra for All"--is not serving anyone well--neither those who fail nor those who pass. Fortunately, good examples show that we can do better: although the challenge of reform is daunting, we know the direction in which improvement lies.

Algebra is, in Bob Moses' apt phrase, a "new civil right." This idea encapsulates the best expression of the motivation for the whole conference--of the source of power that underlies the concept of "Algebra for All." It encompasses a host of powerful ideas:

- Algebra can democratize access to big ideas.
- Algebra provides evidence of the power of one's own mind--an unparalleled source of authority for youth.
- Tracking, the chief alternative to "algebra for all," serves primarily as an engine of inequity.
- Effective instruction requires a respect for diversity grounded in multicultural perspectives.

A second major theme that I heard reflected in virtually all working groups is that algebra in school should serve broad purposes, not just job-specific skills. Its goal is not to convey the contents of a formulary, but to help students acquire the capacity to learn new things. Under this theme I group a number of common conference understandings:

- Teach a few big ideas well, not many superficially.
- In order for algebra to serve broad purposes it is essential that we develop and value diverse ways through which students can succeed with algebra.
- Algebra should be viewed not as a particular course, as it most often is, but as a set of ideas, skills, and habits of thought. Indeed, Algebra should be a strand everywhere throughout K-16, not just a separate course.
- Typical college courses fail to model good instruction, thus sending improper signals to students, schools, and prospective teachers.
- Examples should precede theory; the specific should come before the general.

Finally, throughout our conversations there ran a counterpoint of idealism versus reality. This led to a third area of consensus: In making recommendations, we must distinguish short-term (politically-constrained) actions from long-term (unconstrained) vision.

The "Story Line" of Algebra

We all agree that algebra is in part about symbols. We also seem to agree that algebra is itself a symbol. There agreement ends. As I listened, I sensed that algebra is used by mathematicians, by teachers, and by the public as a symbol--sometimes as a codeword--for a number of different political and social agendas. Some examples:

- What parents studied in school--the embalmed memories of a bygone era
- The contents of the algebra book--engraved authority
- Job screening--a hurdle for quality employment
- Requirement for graduation--a mark of rigorous education
- Key to good jobs--the economic metaphor
- Motivator for high expectations--algebra as the epitome of achievement
- Symbol of failure and phobia--the worst memories of school mathematics
- Filter for success--a hurdle to ensure stability of the ruling class

Yesterday Bob Moses asked for the "story line" of algebra. What can we say to inquiring skeptics to convince them that algebra really is important? Yesterday no one dared take up the challenge. We owe Bob--and ourselves--an answer. So let's provide it now: I'll stop talking for three minutes while each of you write 2-3 sentences on why you believe that algebra is so important. Pick one of two options according to your interests:

A. Why algebra for all? (Addressed to skeptical parents or school board members.)
B. Why take abstract algebra? (Addressed to a college student or a prospective high school teacher.)

[*Note*: An edited compilation of the responses appears in Appendix A.]

Various speakers promised to tell us, at last, just what algebra is. None did. However, I found in my notes three categories of responses. Some thought in traditional terms such as

Logic and proof	Symbol Sense
Use of variables	Simple manipulations
Solving equations	Modeling
Deductive reasoning	

Others spoke in more abstract categories such as

Generalized arithmetic	Means to solve problems
Study of relationships	Study of structure

Henry Pollak challenged us to think about new connections of algebra

To spreadsheets	To data analysis
To stability	To algorithms

These divergent perspectives may help explain why we have found it so difficult to define the essence of algebra--why we have so often avoided the very topic of this workshop. As the elephant to the blind man, algebra appears very different to its many different constituencies.

Language and Understanding

One of the more fascinating recurring themes of this workshop has been the various speculations about the relation of algebra to language. It began with Victor Katz's many historical examples of algebra problems posed and solved entirely in natural language. This strand continued with Mike Artin's ruminations about the relations of language, abstraction, numbers, and algebra.

Alba Thompson continued this theme by showing through many examples the consequences of disjuncture of words from symbols. Many students see no meaning in symbols, and their teachers frequently cannot convey meaning without using the very symbols that their students do not understand. Example: Why does $0 \bullet 3 = 0$? How can this be explained to a primary school student? Will the typical university course in abstract algebra help convey to the prospective teacher an explanation that a young student can understand?

Discourse about algebra is the key to keeping alive a capacity for reasoning and proof. Ten words that students understand may be worth a thousand meaningless symbols.

Algebra in the Workplace

A recurrent theme at this workshop is the changing role of mathematics in the workplace. Today's schools and curriculum are the legacy of an era in which shopkeeper arithmetic was the mainstay of workplace requirements for mathematics. "Algebra for all" is motivated in part by the assumption that algebra is the key to well-paying jobs (for individuals) and to a competitive workforce (for society). Several themes emerged in these discussions:

- Most mathematics used in work is just high-school-level mathematics. Very few individuals use college-level (calculus-based) mathematics in work situations.
- The curricular focus for workforce issues is on the upper two years of high school--in Algebra II and its twelfth grade successor. This is where the "school-to-work" and "tech-prep" themes are most noticeable. One must distinguish at this level three different intentions: pre-work, pre-college, and pre-calculus.
- Algebra is an "invisible culture" not recognized in the workplace. Neither employers nor employees are aware of the extent to which problem solving skills honed in algebra are used in everyday workplace situations. It takes an attentive observer to see the role played by mathematics in ordinary work.
- Nevertheless, lots of good algebra can be found in the workplace, although little of it appears in the classroom. For many reasons--textbook tradition, teacher background, pedagogical difficulties--the "applications" of mathematics continue to be presented in isolation from the context in which they arise.
- Traditional curricula--particularly school algebra texts--teach mathematics first, then applications. The rationale is to first master the tools, then apply them to use. Modeling reverses that order: the problem context leads to the mathematics. A modeling approach, which better fits workplace needs, can clash with traditional curriculum since one cannot provide advance assurance of which mathematics will be covered or learned.

<u>College and University</u>

Whether because it is my natural interest or because it is just such a complex and important issue, the role of algebra in college seemed to command a lot of attention, especially in the working group sessions. I heard four broad persistent worries in these discussions:

A. The nature of high-school-level algebra when taken in college--frequently by adults;
B. The need for sweeping reform in undergraduate mathematics in order to improve the preparation of teachers;
C. The "brick wall" students encounter in the first course in abstract algebra; and
D. The evolving demands for tighter coupling between college-level mathematical programs and the world of work--especially in the two-year colleges.

First a bit of terminology. By "high school algebra" I mean to include all the algebra that leads up to calculus, including the contents of the course that traditionally has been called "college algebra." This is, roughly, the algebra of the eighteenth and nineteenth centuries. By "college-level algebra" I mean to include linear algebra and abstract algebra--courses normally taken following calculus that form the heart of an undergraduate mathematics major. One of the major problems of algebra as it is practiced in today's schools is the lack of mathematical, pedagogical, and psychological connection between these two kinds of algebra--between the pre- and post-Noether views of the subject.

Several issues revolve around high school algebra:

-- If high schools did their jobs well, three-quarters of all mathematics courses in community colleges would need to be totally redesigned. This is not an argument for maintaining the status quo--in which community colleges largely repeat the high school curriculum--but a reflection of the need to develop imaginative post-high school courses that meet the real needs of students for broader mathematics education.
-- Colleges and universities do not give credit for "remedial" courses, so students take these courses under a financial and psychological handicap. What are the educational reasons for denying college credit to Algebra I while awarding it to French I?
-- Most college faculty don't want to teach students who enter college still in need of high school algebra, nor do they give much of their leadership energy to making courses for such students effective.
-- We need a new suite of post-Algebra II courses, especially in community colleges, that provide algebra-based materials in the service of major vocational programs (health, business, ...).
-- Placement testing in colleges and national exams taken by college-bound students impede innovation in high school algebra by establishing public expectations for the traditional curriculum among students, parents, and teachers.

I am, reluctantly, focusing this summary on issues other than teacher preparation. I do want, however, to make one suggestion--actually, more like a challenge--about the need for rethinking the mathematics that is offered to prospective elementary school teachers. Suppose, as we hope for the future, that students enter college with a mathematical experience similar to that suggested by the NCTM *Standards*. We would then need a new course that might be thought of as "elementary mathematics from an advanced standpoint," or perhaps as "advanced mathematics from an elementary point of view." Such a course could serve well the mathematical needs of prospective elementary teachers; of future parents who need to understand mathematics in order to support their children's education; and of educated citizens who are seeking the benefits of a liberal arts education. These three constituencies share much in common--the need for deep understanding of elementary mathematics--as distinct from the professional who needs broad understanding of many parts of advanced mathematics. I suspect that a new course, perhaps incorporating themes similar to those in *On the Shoulders of Giants*, could serve all three purposes well.

The Brick Wall

Faculty in departments of mathematics have always known that a student's first encounter with abstract algebra --"real mathematics"--is often a traumatic experience. Abstract algebra and to a lesser degree the parallel course in elementary real analysis are often the first college courses that employ proof in an essential way. Students whose view of mathematics is shaped by two years of problems-oriented calculus enter these junior-level courses with a distorted view of the nature of mathematics. For many, they hit a brick wall and never recover.

Conferees who have had extensive experience with this phenomenon report that the situation is much worse now than it was, say, twenty years ago. Various explanations are offered. Students have changed--the population is more diverse, with more diverse aspirations and goals; calculus has changed--to emphasize skills and problems, not rigorous thinking; high schools have changed--to stress preparation for calculus rather than preparation for mathematical thinking. Probably all conjectures are valid: today's students who enter college-level algebra courses are expected to abstract from things they haven't experienced--thus violating a common lesson of both history and pedagogy.

One conjecture emerged that intrigues me. I pass it on for your consideration, not as a definitive explanation, but as a way to view the problem that may harbor clues to possible solution.

The route from arithmetic to calculus is through high school algebra. During the past two decades there has been a clear shift in emphasis, reinforced in recent years by the introduction of graphing calculators, to emphasize functions--that is, pre-analysis--rather than operations--that is, pre-algebra. Does the "functions" approach to high school algebra impede development of key algebraic skills and intuition?

The following schematic diagram suggests two possible mathematical routes from grade school to college:

Grade	Pre-Analysis Route	Pre-Algebra Route
K-6	Arithmetic	Arithmetic Operations
5-8	Quantitative Measure	Algorithms & Advanced Operations
9-11	Functional Approach	Algebraic Approach
11-13	Elementary Functions	[???]
12-14	Calculus	Linear Algebra
14-16	Analysis	Abstract Algebra

The point of this chart is to suggest that the natural bridge to calculus provided by the pre-calculus emphasis in the upper years of high school does not exist for abstract algebra. Much of high school algebra now serves to introduce students to the function zoo so that they enter calculus with a rich base of examples on which to build abstract ideas. Where in the curriculum do students encounter a similar base of algebra-rich examples from which to build the abstractions required for modern algebra?

Big Ideas

Everyone seems to agree that algebra should focus on a few big ideas rather than lots of isolated skills. Part of the "avoiding algebra" subtext of this workshop has been the powerful reluctance of everyone to avoid proposing these key ideas. The explanation for this reluctance can hardly be an attack of modesty, since opinions have flown freely on many subjects on which we are in many ways less expert. It is hard to believe that no one has any idea about the key ideas in algebra.

Maybe the problem is that no one wants to go first. Well, now that it is less than an hour to go before the close of the workshop, this is our last chance to address the issue. I'll stop for five minutes to let each table discuss this question: What are the few key ideas of algebra that must be addressed in any curriculum? Each table will make a list--keep it short, please--and we'll see afterwards if there is any consensus. [Note: Appendix B contains the list of "key ideas" produced by the conferees through this exercise.]

Debates and Open Questions

Throughout the workshop I picked up hints of debates, disagreements, and areas requiring further investigation. Here are a few samples, enough to suggest that there is plenty of work ahead for all of us:

1. Does the study of algebra really develop general problem solving skills? Was its historical use to empower the elite based on its real utility in developing broad political problem-solving skills, or might the persistence of algebraic hurdles for leadership been merely a convenient social filter designed to select those who had the advantage of a good education?
2. Nearly everyone at this meeting seems to support the assertion that the same "algebra for all" in grades K-11 will serve equally well college-bound and work-bound students. Although the pace and depth of learning will differ from student to student, the core conjecture of "algebra for all" is that the *same* algebra is best for all, at least up to age 16 or so. Question: Is this an article of faith, or a claim rooted in evidence? We definitely need to learn more about how mathematical thinking is used on the job to determine whether it is defensible to adhere to a common core up until the 11th grade. (Of course, earlier fragmentation would re-introduce tracking, with all its negative social consequences.)
3. “We need to understand better the nature of student understanding of algebraic concepts.” This is what I might call my Alan Schoenfeld memorial sentence, since he wisely asserted it with increasing frequency as discussions seemed to drift into areas in which our ignorance exceeds our knowledge. I'd like to propose an iteration: “We need to understand better the nature of faculty understanding of student understanding of algebraic concepts.” (I'm sure that John Conway can find a way to

provide several more iterations.)

4. Is "watering down" a code word for preserving the filter role of algebra? Is the public pressure for "excellence" just a subterfuge for gaining an advantage--excellence for the elite, mediocrity for the masses? There is some evidence that parental pressure for maintaining traditional courses validated by traditional exams is more a means of maintaining a competition which they believe their children can win than it is a true belief in the educational wisdom of these exams. Similarly, some university faculty insist on traditional entrance expectations not so much because they have concluded based on a review of all the evidence that these expectations yield better education, but because they fit better the courses the faculty are accustomed to teaching. There is a real danger that any kind of "new" algebra will be rejected either by the public or by mathematicians simply because it changes the rules of the game.
5. Is there a better approach to elementary algebra for adults than repeating prior failure? Must everyone repeat the traditional syllabus, regardless of age, background, and educational objectives? Algebra is no longer just a subject taught to adolescents; more adults study high school algebra than study calculus and statistics combined. We need to find better ways to help everyone learn algebra.
6. Related to the previous question is one of a narrower but nonetheless fundamental character: Can one delay symbol manipulations (in Algebra I) and then catch up later? If symbol manipulation is now less important for real applications, can it be delayed without serious harm to students' ability to learn subsequent algebra and abstract mathematics? Is there an appropriate time for learning algebra--as there is for language--that, once passed, is difficult to recover?
7. Some people have been discussing a political and rhetorical issue: Do we want to stress "algebra for all" or quantitative literacy ("numeracy")? The former may have the flavor of medicine, the latter of a civic duty. What are the political implications? Voices were heard on all sides of this issue.
8. Finally, a question based on reflection from many side conversations: Is an emphasis on what I might gently term "revolutionary" ideas (e.g., eliminating all algebra courses in school; utopian use of technology) just a way to avoid the real problems of today's schools and colleges? This perennial tension between idealism and pragmatism was well reflected by the workshop participants--and in our discussions. It is a problem that we all face everyday in our work, one that we must bear in mind as we seek public support for improved mathematics education.

<u>Challenges</u>

I close with several challenges that arose from many parts of this workshop:

1. Can we create a rich curriculum for high school algebra that truly integrates workforce and academic perspectives?
2. Can technology help develop algebraic and reasoning--not merely geometric--skills? Can there be an "algebraist's sketchpad"?
3. Can the clients of high school algebra be persuaded to forego any part of the traditional repertoire of symbol manipulation? Can students learn mathematics if

manipulations are slighted?

4. Can teachers possibly keep up with the proposed changes? If we believe the popular diagnosis about weak preparation of teachers for today's curriculum, how can we imagine that they could teach a curriculum based on some of the more imaginative ideas at this conference? Could the U.S. afford the required retraining?
5. Can we persuade the public that new algebra is real algebra? How can we allay the suspicion that algebra without mind-numbing manipulation is not just another example of "dumbed- down" curriculum?
6. Can mathematicians retain control of algebra in the face of the new proposals for national certification of standards and performance expectations, e.g., the proposed National Education Standards and Improvement Council (NESIC)? Will the symbolism of algebra as a public good shift control from professionals to politicians in setting national standards? Who, ultimately, will decide what algebra is?

Gleanings and Grazings

Hidden here and there in our conversations and speeches are a variety of slogans that I leave as my parting contribution:

-- Algebra before acne
-- Algebraists Anonymous
-- Algebra as a new civil right
-- A "Declaration of Ignorance"
-- More Math Means More Money
-- Those who can, do; those who understand, teach.
-- Number Sense -- Symbol Sense -- Function Sense
-- 6 R's of College: Remedial Reading, Remedial Writing, Remedial 'Rithmetic
-- Quantitative reasoning is algebra without symbols, algebra without knowing it.
-- “Algebra for all” is like a slow motion train wreck: if it fails, it will confirm the public belief that most students can't learn algebra

Appendix A:

Why Study Algebra?
Conferees' Perspectives

Why require algebra? (Addressed to parents and school boards)

- The information age is characterized by the generation of massive quantities of data; indeed, "information" is often identified as "the new capital." Increasingly, the ability to deal with large bodies of data is critical both to employability and to the exercise of the responsibilities and rights of citizenship. In order to do this, it is essential to consider variables within the data, and to view the data by imposing (or finding) structures on it. Algebra is the science of variables and models; knowledge of algebra is therefore the key to both employment and citizenship.
- Algebra helps develop students' ability to conceptualize ideas from the specific to the general. It helps them discover patterns among items in a set. Finally, it enables them to think through the various aspects of a problem-situation, to identify known and unknown quantities and the relationships among them, and to develop a strategy for determining the values of unknown quantities.
- Mathematics in all its facets provides ways of organizing, making sense of, and reasoning about patterns in the world of our experience. Many of the most significant patterns involve relations among variable quantities. Algebra is the basic set of ideas and techniques for describing and reasoning about relations among quantitative variables.
- Algebra is a powerful way to organize quantitative relationships which in turn can reveal new information ("solutions"). With an algebraic underpinning, one can see fundamental similarities in structure among diverse contexts.
- Algebra provides students with the knowledge and ability to manipulate situations (symbols) so that they may better understand what is going on around them in the world.
- Algebra provides a language and way of thinking which allows us to think about and communicate to others general properties and patterns of situations involving uncertainty, change, quantities, size, spaces.
- Students need the general mental discipline and specific problem solving skills of algebra (e.g., manipulating symbols, solving for the unknown in order to compete for and hold an increasing number of technologically-related jobs in our economy.
- Ours is a technological society and becoming increasingly so. Algebraic thinking, the experience of looking quantitatively at situations and data (not the use per se of variables) is necessary to be a productive, contributing citizen, both through employment and consumerism.
- Algebra is the entry point for formalism and abstraction. It is required in order to understand the world around us and to function effectively in it as a citizen and worker. The nature of work and of jobs more and more requires the capacity for formalism and abstraction to formulate issues and to solve problems.
- Algebra is the one place in the high school curriculum where students are taught how to reason quantitatively and objectively about situations in the real world where the quantification cannot just be obtained by common sense and guessing. Elements include symbolic representation, precise computation, and interpretation of the symbolic (or numeric) variables in the context of the problem--in short (mathematical) modeling.
- Algebra is becoming a universal language which describes relationships--covariation between two or

more traits in many disciplines, businesses, and discussions for citizenship.

- Most of contemporary society, our technology, and our work depends upon mathematical models. Those models require an understanding of algebra. The ability to use algebra in constructing these models is essential to success in today's world.
- We need an algebra program that everyone should have. In it, students would learn to understand events and systems that are important in their work and in society more analytically and precisely in terms of their quantitative aspects; they would also learn to recognize and formulate key questions and learn to formulate arguments more persuasively.
- Algebra is a language one uses to represent the relationships of one set of variables to another set. The representation process involves tabular, graphical and symbolic symbols and rules. It is the language of mathematics, science, business Therefore all should learn to use it.
- By being successful in this "new" algebra, a student will explore relationships among quantities and ways of thinking mathematically which describe life in our complex world and the workplace.
- Algebraic thinking enables on to figure things out by thinking in terms of what's known, what's not known, and what are the relationships among them.
- Algebra is the "new literacy." 75% of jobs require Algebra I and some geometry Therefore, to be prepared for society (and jobs), students need algebra.

Why study algebra? (Addressed to high school students)

- Properly learned, algebra embodies both an analytic perspective--you learn to make sense of situations, to find out "what makes things tick"--and a symbolic language that helps to represent such situations and make sense of them. It's an important aspect of "mathematical power."
- Algebra at the high school level has two components. One might be called ideas (or concepts), and the other means of representation. Concepts include, for example, the distinction between linear and non-linear phenomena, the idea of rate of change and the rate of change of the rate of change. Means of representation are used both to communicate with others, and to think about things ourselves. They include graphs, tables, finite differences, algorithms, recursion rules, and expressions like y = e^(-t) sin(alpha)t.
- We are going to learn to analyze situations, that is, to understand what is going on, and look for the principles of what is going on, and therefore what should be done. We will pull out ways of thinking that can be used elsewhere.
- Algebra introduces way of modelling (describing) real world situations. It involves use of symbols and graphs, and techniques for solving problems and making inferences. For example, y = 2(x-60) + 95 is model that yields the speeding fine on the Pennsylvania Turnpike.
- I would like you to travel to a town that you have never visited. When you get off the plane I want you to imagine that all signs have been made invisible to you, but everyone else can read them. Your job is to get to your hotel. You can well imagine how lost you would be. The same is true of anyone who is ignorant of mathematics as that person tries to make his or her way through nature.
- Algebra is the first stop beyond arithmetic in quantitative thinking. It helps students learn to formulate and solve problems, identify and communicate patterns and trends. It builds skills and patterns of thought essential to accomplishing many quantitative challenges we face in everyday life (home finances, business, trade, professions, etc. Indeed, it is fundamental to quantitative literacy.

- To be able to deal with problem situations that arise in life without algebra as a tool for reasoning would be very difficult, if not impossible. Algebra provides the language that is used in the study of more mathematics.
- Through the study of algebra you will learn the skills of abstraction and generalization that are indispensable for operating in the information age.

Why study algebra? (Addressed to college students)

- Most mathematics students in college still think of mathematics as problem-solving and computation. More algebra taught well gives them a glimpse of systematic thinking, of the essence of mathematics. The skills of advanced algebra are generalizable to computing, to logic and to higher order reasoning. It opens intellectual and not just career doors.
- Algebra is a branch of mathematics which is essential to main areas of physics. It also has applications for the recently developed areas of error-correcting codes and cryptography. It is the language of mathematics and part of our cultural heritage.
- Algebra studies the commonalities that occur in various number systems or other objects and operations on them (matrices, transformations, logical propositions). The study of algebra shows the power of mathematics to synthesize a variety of mathematical properties from different contexts.
- Abstract algebra is the study of the structure of sets equipped with formal operations. By abstracting the essential ideas of structure, we are able to see the common features of diverse situations. Rings, for example, complete the structure of arithmetic and polynomial manipulation of high school algebra.
- Linear algebra provides a means for manipulating data. It is a means to solve systems of equations. It also provides a tool for expressing and solving geometrical problems. It is a course that has connections to many mathematical and geometric ideas.
- Linear algebra is the story of linear relationships. It started off with the complete theory of systems of linear equations, but has gone far beyond that. Often something in the real world is specified by a number of parameters--numbers like x, y, z, ... Some other facts about it are expressed by linear functions of these parameters, like

  $X = 3x + 5y - 7z \ldots$

  $Y = 2x - 9y + z \ldots$

  Linear algebra studies these linear relationships. You might have a reactor vessel, with pressure, temperature, concentrations as the starting numbers x, y, z, ... and reaction products as the new ones, X, Y, Z, ... Or it might be some other very different situation. There is a rich and powerful and non-obvious theory of all this: it's called "linear algebra." In addition to "real life" examples, there are lots of other examples inside mathematics itself. Indeed linear algebra is fundamental to almost all pure mathematics as well as being the most prominent type of mathematics used in its applications.
- Why study higher algebra? In order to see the power of abstract ideas, for example, groups, which one can get by abstracting from several examples (the integers, permutations), feeding back to apply to number theory and combinatorics (Burnside enumeration, chemical molecules) to give insight into many other systems.

Appendix B

A Few "Big Ideas" of Algebra
Conferees' perspectives

- Generality, representation, quantity, structure, relationships.
- The use of symbols to represent ideas or numbers
- The expression of relationships by equations
- The way expressions vary as the numbers in them change
- The difference between linear and non-linear behavior

- Representational abilities for modelling systems with variability
- Operations on systems focusing in common properties, structures and functions
- Power of symbols to assist reasoning and to communicate
- Modeling--transition from physical to mathematical

- Reasoning about quantitative situations
- Recognizing patterns and important (basic) functions
- Reasoning with these patterns
- Communicating ideas about these patterns in multiple representations: verbal, tabular, symbolic, graphic, and in problem situations.
- Linearity vs. non-linearity
- Equivalent representations (within and among graphs, tables, symbols)

- Variable quantities (x)
- Representation: numerical, graphic, verbal, symbolic
- Operations
- Structures
- Functions and Relations
- Symbol Sense
- Generality

- Representational systems
- Linearity and non-linearity
- Relationships and functions
- Structure
- Solving systems
- Abstraction
- Algorithm (Proof)

- Symbol sense
- Patterns, descriptions
- Symbol manipulation
- Obtaining unknown information from known
- Multiple representation
- Underlying structure