Point of View *By Lynn Arthur Steen*

Who Still Does Math with Paper and Pencil?

New computers and pocket calculators are about to sweep the campuses; as a result, ground rules for teaching must change

ILLUSTRATION FOR THE CHRONICLE BY MAX-KARL WINKLER

MATHEMATICS is now so widely used in so many different fields that it has become the most populated—but not the most popular—undergraduate subject. Each term, an army of three million students labors with primitive tools to master the art of digging and filling intellectual ditches: Instead of using shovels and pick axes, these students use paper and pencil to perform millions of repetitive calculations in algebra, calculus, and statistics. Mathematics, the queen of the sciences, has become the serf of the curriculum.

People who use mathematics in the work place—accountants, engineers, and scientists, for example—rarely use paper and pencil anymore, certainly not for significant or complex computations. Electronic spreadsheets, numerical-analysis packages, symbolic-algebra systems, and sophisticated computer graphics have become the power tools of mathematics in industry. Even research mathematicians use computers to help them with exploration, conjecture, and proof. In the college classroom, however, mathematics has—with few exceptions—remained in the paper-and-pencil era.

Academic inertia alone is not a sufficient explanation for this state of affairs. Other disciplines—notably chemistry, economics, and physics—have adapted their undergraduate curricula to include appropriate use of computers.

In contrast, many mathematicians believe that computers are rarely appropriate for mathematics instruction; theirs is a world of mental insight and abstract constructions, not of mechanical calculation or concrete representation. Most mathematicians, after all, chose mathematics at least in part because it depends only on the power of the mind rather than on a variety of computational contrivances.

All that is going to change in the next two or three years, which in education are the equivalent of a twinkling of an eye. The latest pocket calculators with computer-like capabilities can perform at the touch of a few buttons many of the laborious calculations taught in the first two years of college mathematics. They can, among other things, graph and solve equations, perform symbolic differentiation as well as numerical and some symbolic integration, manipulate matrices, and solve simultaneous equations. Although such computations do not form the heart of the *ideal* curriculum as it exists in the eye of the mathematician, they do account for the preponderance of the *achieved* curriculum that is actually mastered by the typical undergraduate.

Computation has become significant for mathematics because of a major change not just in scale but in methods: the transition from numerical mathematics, the province of scientists, to symbolic and visual mathematics, the province of mathematicians. Large computers have been doing "real" mathematics for years, but cost and relative scarcity kept them out of the classroom. No more. Mathematics-speaking machines are about to sweep the campuses, embodied both as computer disks and as pocket calculators. Beginning this fall, college students will be able to use such devices to find the answers to most of the homework they are assigned.

Much as professors like to believe that education standards are set by the faculty, the ready availability of powerful computers will enable students to set new ground rules for college mathematics. Template exercises and mimicry mathematics—staples of today's texts—will vanish under the assault of computers that specialize in mimicry. Teachers will be forced to change their approach and their assignments. They will no longer be able to teach as they were taught in the paper-and-pencil era.

CHANGE ALWAYS INVOLVES RISK as well as benefit. We have no precedents for learning in the presence of mathematics-speaking calculators. No one knows how much "patterning" with paper-and-pencil methods is essential to provide a foundation for subsequent abstractions. Preliminary research suggests that it may not be as necessary as many mathematics teachers would like to believe. On the other hand, many students tolerate (and survive) mathematics courses only because they can get by with mastery of routine, imitative techniques. A mathematics course not built on the comfortable foundation of mindless calculation would almost surely be too difficult for the student whose sole reason for taking mathematics is that it is required.

Despite such risks, mathematics—and society—has much to gain from the increasing use of pocket computers in college classes:

- Undergraduate mathematics will become more like real mathematics both in the industrial work place and in academic research. By using machines to expedite calculations, students can experience mathematics as it really is—as a tentative, exploratory discipline in which risks and failures yield clues to success. Computers change our perceptions of what is possible and what is valuable. Even for unsophisticated users, computers can rearrange the balance between "working" and "thinking" in mathematics.
- Weakness in algebra skills will no longer prevent students from pursuing studies that require college mathematics. Just as spelling checkers have enabled writers to express ideas without the psychological block of worrying about their spelling, so the new calculators will enable students weak in algebra or trigonometry to persevere in calculus or statistics. Computers could be the democratizer of college mathematics.
- Mathematics learning will become more active and hence more effective. By carrying most of the computational burden of mathematics homework, computers will enable students to explore a wider variety of examples, to study graphs of a quantity and variety unavailable with pencil-and-paper methods, to witness the dynamic nature of mathematical processes, and to engage realistic applications using typical—not oversimplified—data.
- Students will be able to explore mathematics on their own, without constant advice from their instructors. Although computers will not compel students to think for themselves, these machines can provide an environment in which student-generated mathematical ideas can thrive.
- Study of mathematics will build long-lasting knowledge, not just short-lived strategies for calculation. Most students take only one or two terms of college mathematics, and quickly forget what little they learned of memorized methods for calculation. Innovative instruction using a new symbiosis of machine calculation and human thinking can shift the balance of mathematical learning toward understanding, insight, and mathematical intuition.

Mathematics-capable calculators pose deep questions for the undergraduate mathematics curriculum. By shifting much of the computational burden from students to machines, they leave a vacuum of time and emphasis in the undergraduate curriculum. No one yet knows what, if anything, will replace paper-and-pencil computation, or whether advanced mathematics can be built on a computer-reliant foundation. What can be said with certainty, however, is that the era of paper-and-pencil mathematics is over.

Lynn Arthur Steen is professor of mathematics at St. Olaf College and chairman-elect of the Conference Board of the Mathematical Sciences.