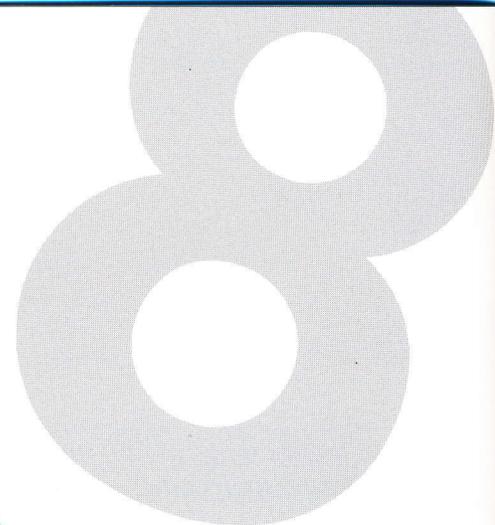


STRESS POINTS IN SCHOOL AND COLLEGE MATHE- MATICS

Out-of-date textbooks, inappropriate tests, and underprepared teachers have left mathematics in the classroom in disarray.



One of the ironies of our age is the striking contrast between mathematics in the marketplace and mathematics in the classroom. In science and industry, mathematics is a thriving activity, supporting through statistics, computing, simulation, and theory the most complex processes of society. But in the classroom it is in disarray, with weaknesses at every level from primary school through graduate school. Although we live in a minds-on world, mathematics in the classroom more often than not turns young minds off.

Last year the National Academy of Sciences released two reports that bear in important ways on the role of mathematics in education. *Renewing U.S. Mathematics* documents, in a chapter called "Ordering the Universe: The Role of Mathematics," the dramatic extent to which mathematics is used in science and industry:

In the past quarter century, mathematics and mathematical techniques have become an integral, pervasive, and essential component of science, technology, and business. In our technically oriented society, "innumeracy" has replaced illiteracy as our principal educational gap. One could compare the contribution of mathematics to our society with needing air and food for life. In fact, we could say that we live in the age of mathematics—that our culture has been "mathematicized."

The other Academy report, *High Schools and the Changing Workplace: The Employer's View*, argues that the school needs of students entering the workplace directly are not much different from those who go to college: "Those who enter the workforce after earning a high school diploma need virtually the same competencies as those going on to college, but have less opportunity or time to acquire them." All need to function in the minds-on world derived from modern mathematical science.

Yet the evidence of recent reports and tests suggests that we are not now preparing students well for the world in which they will live and work. Society, especially the workplace, has changed far more rapidly than have curriculums or teachers. The inertia of the educational system is so great that over the years it accumulated, with barely any public notice, an enormous burden of struc-

tural debt. Now that the public has finally noticed, we must face the realities of a system under great stress.

The first point of stress is the sad fact that most of the mathematics taught in United States colleges and universities is really high school mathematics. In every country in the world except the United States, school mathematics ordinarily includes a one-year introduction to calculus; not all students take such a course, but if they do take it, it is usually part of the preuniversity curriculum. By this world-standard definition, nearly 90 percent of all United States mathematics course enrollments in postsecondary education are in high school mathematics. Even if we adopt the less stringent definition that identifies calculus as part of the postsecondary curriculum, we find that two-thirds of the mathematics taught in colleges is really high school mathematics.

Traditionally, high school mathematics has followed a rather well-defined route with only minor side excursions: algebra I, geometry, algebra II, precalculus, calculus. Twenty percent of the college-bound students begin the sequence in eighth grade and complete it by grade 12; the others begin it in ninth grade and complete it, if at all, during college.

Despite the handicap of rigidity, the regularity of this curriculum has been generally beneficial. Students who move often in our mobile society can adjust reasonably well to the mathematics curriculum anywhere in the United States. The sequence fits well the linear nature of mathematical learning, at least for those who are moving towards calculus. To the extent that the traditional curriculum is effective, it insures a common base of knowledge for the workplace, a general quantitative literacy appropriate for civic responsibility, and suitable background for business, science, mathematics, and engineering courses in college.

In the last two decades, however, the role of mathematics in science and industry has changed dramatically. The mathematical sciences, as they are now called, encompass such diverse areas as data analysis, computing, and operations research. At the same time, computers have entered the workplace, dramatically altering our perspective on which parts of mathematics are really fundamental and which are not. The result is an increasing degree of chaos in which schools respond to diverse

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demands to add newly-important parts of mathematics to an already over-filled curriculum. Ill-focused demand for curriculum reform places major stress on school teachers and administrators.

The dominant pressure from the public is for courses in computing, a broad term covering everything from word processing to the theory of algorithms. It is very important for curriculum planners to distinguish between different aspects of computing in school curriculums. Some courses teach students to use programs (word processing, spread sheets), while others teach students to write programs (usually Basic, Logo, or Pascal); some teach about the role of computers in society (information technology, computer crime, industrial robots), whereas others teach about the nature of computer science (algorithms, data structures).

The diversity of computer courses in high school causes yet another stress on mathematics education because computer courses often substitute for mathematics courses in the high school curriculum. Already several states and many districts have set rules that classify any course using computers as part of the hours required for mathematics. In reaction, the National Council of Teachers of Mathematics has just adopted a policy statement opposing the inclusion of *any* computer course for school requirements in mathematics. Students need both computing and mathematics—not one or the other.

The pressure to learn subjects appropriate for the computer age leads to many suggestions for change in the mathematics curriculum itself. Computers make it possible to do things that could never be done before, such as deal with real data in the classroom and picture dynamically the functions studied in school mathematics. Using the computer in this fashion, as a natural tool for exploring mathematics, has tremendous potential for improving both the relevance and the appeal of traditional school mathematics.

Computers also make it necessary to teach some things that heretofore were not considered essential. Study of what is now called “discrete” mathematics has become very popular in college, and some topics from this new field are infiltrating the school curriculum as

well. Discrete mathematics is essential for the study of computer science. Interestingly, many of the topics essential to discrete mathematics are the fundamentals of the “new math” in modern dress: sets, relations, binary numbers, Boolean algebras. Other topics such as graph theory are truly new. One new theme, essential to both computing and mathematics, is the study of algorithms—the step-by-step procedures for solving problems that provide the intellectual basis for computer programs. Somehow the mathematics curriculum must make room for all these new topics while at the same time teaching the traditional topics from school algebra and geometry.

The greatest controversy in the mathematics curriculum caused by the introduction of computers derives from the simple observation that computers render irrelevant some things that formerly were viewed as fundamental. Calculators did the same thing. How many adults, whether store clerks or scientists, ever do long division by paper and pencil? Similar questions are now being asked about many parts of standard school mathematics. Computers can solve equations, differentiate functions, and multiply matrices far more accurately and more rapidly than people can.

So here is another stress point in the mathematics curriculum: Should we continue to teach students to carry out by paper and pencil the many calculations that computers can now do much better? Is it our aim to make students into back-up computers, or is there some other justification for all the calculations that make up so much of school mathematics? On the other hand, if we reduce the emphasis on routine calculations, will students have sufficient practice to learn to comprehend and solve complex problems?

Curricular response to the impact of computers has been slow in all these areas: We are not taking full advantage of what computers make possible, nor are we introducing topics that computers make necessary, nor are we excising habits that computers have made obsolete. The educational system maintains a steady course irrespective of external pressure due to the unchanging

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guidance of three curricular gyroscopes: teachers, textbooks, and tests. Each of these imposes serious stress on school mathematics.

United States mathematics classrooms are burdened today with large numbers of teachers who are seriously underprepared to teach a contemporary mathematics curriculum. Many are older and have been given no serious sustained opportunity for professional development since graduating from college in a precomputer age. Others are recent recruits whose primary fields of study were outside mathematics and whose preparation in mathematics is very skimpy. Of course there are also many teachers who are superbly trained, enthusiastic, and effective. But these master teachers are carrying the burden of the entire mathematical curriculum alone.

Textbooks, by and large, are usually a decade out of date. The effort involved in writing, the time required for production, and the infrequent purchase by school districts all conspire to guarantee that most students are taught from textbooks whose structure was conceived about the time they were born. Yet despite popular opinion, mathematics is not a static subject: It is growing rapidly in methodology, scope, and application. Texts that do not reflect this growth institutionalize both a precomputer curriculum and an incorrect view of mathematics as a static, completed subject.

To a very large extent teachers teach and students study for tests. And standardized tests, like texts, tend to be perpetually out-of-date because of the institutional constraints involved in their preparation. But what is worse, tests always overemphasize computational skills that are now seen as more appropriate to computers, and de-emphasize open-ended problem solving based on formulating conjectures, estimating results, and selecting relevant information. By rewarding unique right answers, tests necessarily examine skills that are more appropriate to computers than to people. In the presence of such tests, it is very difficult for even well-intentioned teachers to move the curriculum in a modern direction.

The school population splits rather early into two groups: those in a precollege curriculum, most of whom take at least algebra II, and those who intend to enter the workforce either directly or via a vocational-technical school. The curriculum for students in the latter group has never been satisfactory: It consists of extended review of the junior high school curriculum disguised under course titles such as business mathematics. Some innovative curriculums (in California and elsewhere) are attempting to establish re-entry courses so that late bloomers can catch up with the precollege track. The National Research Council (NRC) recommendation on school requirements for students entering the workforce suggests that we need a lot of imaginative thought for these courses that will quite likely be the last mathematics courses such students ever take.

Thomas Jefferson spoke of an "enlightened citizenry" as the only proper foundation for democracy. This ideal has sustained public education in the United States on a broader level than is typical in most other countries. Consequently, the recent reports on weakness in science and mathematics education have motivated many school jurisdictions (districts, legislatures, universities) to specify required levels of mathematics for school or college degrees. The result is yet more strain on mathematics education.

Typically, standards for quantitative literacy that become part of college degree requirements necessarily amount to a selection of material from junior high school mathematics and computer programming. Higher standards cannot be imposed uniformly without forcing too many students to fail. But low standards undermine efforts to develop innovative courses in mathematics literacy, substituting instead a deadly review of arithmetic and elementary algebra that rarely produces a citizen enlightened about the role of mathematics in society.

This problem is a stress point not just for school mathematics, but for society in general: Voters unable to appreciate the ways in which figures can lie will be easily misled in their decisions about such important issues as consumer safety, defense policy, and economic
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conditions. We must find a better way to educate students who leave mathematics at an early stage of the curriculum, although not necessarily at an early age. They need to be taught in a way that will build confidence in their own abilities to ask the right questions and to be skeptical of disingenuous answers. They must not succumb to the belief that mathematics is a new form of magic.

When students enter college without the solid knowledge of three years of high school mathematics, they almost always have trouble in some college courses. The economic and psychic costs of "remediation" are very high, and the success rates very low. Most two-year institutions and many comprehensive universities are forced to reteach junior high school mathematics to large numbers of entering students. This practice produces so much educational and social stress as to disrupt what it is intended to facilitate—the student's general education.

Indeed, the greatest stress on college departments of mathematics is the enormous diversity of entering students, together with the increasing diversity of service courses required by other departments. It is not unusual to find freshmen placed somewhere in a vertical tier of 8 to 10 levels of entering mathematics, ranging from "arithmetic for college students" to advanced calculus. Moreover, students can enter college mathematics by selecting from among a broad horizontal spectrum of beginning courses including calculus, discrete mathematics, finite mathematics, statistics, computer programming, computer science, and mathematics appreciation.

This enormous variety of entering courses poses several strains on college faculty. The advising and counseling workload becomes heavy, or if neglected then the dropout rate becomes excessive. Staff loads are frequently very high because of the large number of offerings, and since so much work is at school rather than college level, faculty interest and morale is often low.

A collateral problem reinforcing this degeneration of college mathematics is a sharp drop in advanced enrollments and majors. Since 1970 the number of mathematics majors has declined by more than 50 percent, as have the enrollments in advanced core mathematics courses. The increasing demand for mathematics in science and engineering has recently created a sharp increase in demand for intermediate service courses, but the number of students who major in mathematics is only slowly beginning to rise.

Since the number of mathematics majors has declined from about 25,000 to under 10,000, the number of qualified teachers for both secondary school and colleges has also declined. Indeed, at the Ph.D. level, there has been a serious triple decline: As the number of doctoral degrees in mathematics declined from a 1972 high of nearly 1,200 to a 1985 figure of about 700, the percentage of these degrees awarded to United States citizens has dropped from nearly 90 percent to under 60 percent. Moreover, the percentage of new Ph.D. degree holders who enter college and university teaching declined as well, by about the same amount.

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As a consequence, last year fewer than 400 new Ph.D. holders entered college and university faculties of mathematical science. Only 75 entered the large number of bachelor's degree institutions. The number of vacancies in these institutions is probably five times as high, although firm data on openings are hard to obtain. It appears certain that we are headed into a period when the non-Ph.D.-granting institutions may once again, as happened 40 years ago, find it almost impossible to attract qualified new faculty in the mathematical sciences.

Shortages of teachers, underprepared students, curricular turmoil—these are serious signs of stress that permeate both school and college mathematics. They are not independent problems, but part of a complex system with complex interactions and hidden feedback loops. For example, as enrollments in advanced courses decline and remedial programs grow, college teaching becomes a less attractive profession, so shortages of college teachers increase. Then, as classes grow and the college curriculum stagnates, fewer students are attracted to school teaching because their college courses fail to provide sufficient incentive or enthusiasm.

It is possible, however, to reverse the impact of this negative feedback with appropriate interventions in the system at crucial points. For example, special projects to encourage faculty development in newer areas of the mathematical sciences (computer science, discrete mathematics) can create enthusiasm among the faculty which ignites student interest, thereby attracting increasing numbers of good students to take advanced courses in mathematics. These increased numbers make possible greater variety of advanced offerings, which in turn will spawn more undergraduate majors, some of whom will become excellent, well-trained school teachers. Their students, in turn will help sustain the momentum of growth when they reach college.

What the nation needs is a mechanism for identifying and encouraging these crucial interventions. And we are now getting just such a mechanism. For the first time, the United States now has a Mathematical Sciences Education Board (MSEB) representing the full spectrum of interest groups involved in mathematics education. This board, chaired by Shirley Hill of the University of Missouri at Kansas City, was officially launched by the NRC at the end of October 1985.

The purpose of the MSEB is to provide a continuing national capability for overview and assessment of mathematical sciences education. It will seek to provide leadership to the nation, coordination among educators, service to institutions, information to the public, advice to governments, and recommendations to educational agencies. During the first year the MSEB will concentrate on school mathematics, emphasizing testing, standards, technology, teacher education, and information. In subsequent years it will join with the research community, represented by the NRC Board on Mathematical Sciences, to examine issues in collegiate mathematics.

The Mathematical Sciences Education Board was established as a response to the many serious problems facing mathematics education K-16. It is clear that in a nation such as ours no single overriding solution is possible: We do not have, nor would we want, a national curriculum with national standards. What we need instead, and what the MSEB hopes to provide, is a permanent national capability to influence the many agencies of society (50 states, 3,000 colleges, 17,000 school districts, 30,000 college mathematics teachers, 100,000 secondary school mathematics teachers, one million elementary school teachers) that are responsible for the mathematics education of our youth. ●