Ever Since Socrates

The Beauty of Doing Mathematics: Three Public Dialogues. Serge Lang. 127 pp. Springer-Verlag, 1985. $19.80.

Math! Encounters with High School Students. Serge Lang. 138 pp. Springer-Verlag, 1985. $19.95.

Ever since Socrates, all true teachers of mathematics have insisted that mathematics is not a spectator sport. To understand, you have to think; and if you think, you will understand.

In these two slim volumes, Paris-born Yale mathematician Serge Lang follows the Socratic tradition of teaching mathematics by asking questions. Here's a typical passage from the third of the three public dialogues in *Beauty*:

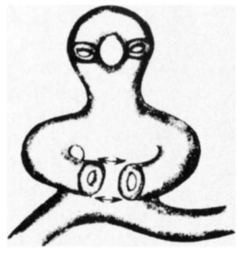
Lang: Can you give me examples of other compact surfaces?
Audience: A cube.
Lang: Yes, but it is equivalent to a sphere. Can you give me an example that is not equivalent?
A Gentleman: A torus.
Someone Else: You make a hole in the sphere.
Lang: Are you a mathematician?
The Gentleman: A little.
Lang: That's already too much.
Someone: A Klein bottle.
Lang: Some of you know too much.
A Child: A pyramid.
Lang: That's equivalent to the sphere.
A Lady: A box without its top?
Lang: Yes, but it will have a boundary. I want a surface without a boundary.
A College Student: Make two holes, like eyeglasses.
Lang: There! That's what I wanted. But are you a mathematician?
Student: Yes.
Lang: Oh no! You are not playing the game. [Laughter]

The game goes on, faithfully recorded, transcribed, and translated from the French originals, based on a series of three interactive public lectures that Lang gave in successive springs at the Palais de la Decouverte (Science Museum) in Paris. The other volume, *Encounters*, records in a similar fashion seven dialogue-lectures given to classes of high school students in Canada and Paris.

Because Lang is doing mathematics rather than merely talking about it, he does not hesitate to use formulas: he plays hardball in these volumes, and only those prepared to play in that league should expect to participate. But for those who are prepared—and that includes everyone who is comfortable with advanced school algebra—these volumes provide a unique window on creative mathematics.

Expositors of mathematics have never achieved either the literary or popular acclaim of writers like Lewis Thomas, Carl Sagan, Jeremy Bernstein, or Stephen Jay Gould who describe the natural sciences. Perhaps that is because mathematics is a mental or formal science, not a "natural" one. If you believe, as do most mathematicians, that understanding requires engagement, then the standard tools of the science writer—description, metaphor, comparison—are of relatively little value in communicating mathematics to the general public. To understand mathematics, there is no substitute for pure thought.

Nevertheless, in recent years there has been a minor renaissance of books attempting to convey the nature of mathematics to lay audiences: the best known, and the best, are *Bridges to Infinity* (Houghton Mifflin, 1983), and three books published by Birkhäuser Boston—*The Mathematical Experience* (1980), *Infinity and the Mind* (1982), and *Mathematical People* (1985). As if to confirm the widespread belief that it is impossible to communicate mathematics to the general public, Birkhäuser Boston recently went bankrupt—although their mathematical publications have been taken over by Springer-Verlag, the house that brought out Lang's two volumes.



Drawings are from *The Beauty of Doing Mathematics.*

The key difference between Lang's efforts and other popular expositions is that Lang really does mathematics. Of the high school students he asks about π: do they really know what it is? All school students know that the area of a circle is proportional to the square of its radius, and that its circumference is proportional to its diameter. They also know that the constants of proportionality in both cases are the same, namely π. Lang asks if they can prove that these two constants are the same: why is the constant in the one formula the same as in the other?

Lang: What is the definition of π? Before you can prove something, you must have a definition.
Student: It's what I said—the circumference divided by the diameter.
Lang: But then you have to show that it is the same π as in the formula for area.

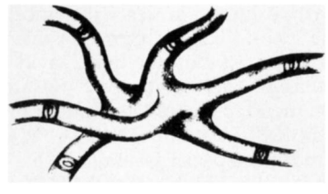
And so it goes, with different classes of high school students pursuing the meaning of area, volume, and circumference, always with an eye on how one knows what one knows.

The high school dialogues in *Encounters* discuss high school mathematics, but from the discerning, disciplined perspective of a mathematician who proves things for a living. The museum dialogues in *Beauty*, on the other hand, plunge forthrightly into contemporary research mathematics, introducing the audiences (and the reader) to the Riemann hypothesis, the Mordell conjecture, and the Poincaré conjecture. In each lecture Lang uses his target conjecture as living proof that the process of mathematical discovery is still going on, and then attempts to draw from the audience sufficient threads to form the warp on which he can weave a tale of mathematical discovery.

One can easily imagine, while reading these dialogues, the vigor of the lecture and of the audience participants. Mathematics teachers will instantly recognize the verisimilitude of misconceptions and confusions in Lang's audiences as they try to think about mathematics. What is unreal, however, for American audiences is the level of sophistication of the dialogues.

The museum lecture on geometry begins with easy concrete dialogue about spheres, cubes, pyramids, and eyeglasses. From such simple beginnings, Lang leads us to this:

Let F be a surface, compact, orientable, without boundary, and not equivalent to the sphere or to the torus. Then there exists a discrete group Γ such that the surface F is equivalent to the hyperbolic plane on which we have identified points with respect to Γ.



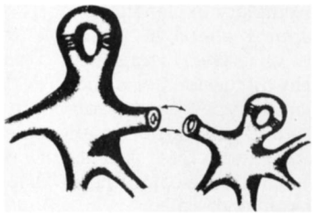
But that's not the end. From this 19th-century result Lang goes on to outline Princeton mathematician William Thurston's very recent insights into generalizations of this theorem to three-dimensional objects. That's heady stuff for an audience of laypersons. Yet it is clear from the recorded dialogue that at least some members of the audience were not

daunted by it. I wonder if American museum-goers would do as well.

The student dialogues in *Encounters* are similarly advanced, but not unrealistic for classes of honors students in any country. There really isn't very much that is unique in them, for good teachers across the country, when they have classes of good students, spend many hours in similar exchanges. The details may differ, but the spirit of good teaching is the same. Partly for this reason, *Encounters* is really of more pedagogical than mathematical interest. Indeed, it concludes with a postscript containing the transcript of a discussion about school mathematics. Here an angry Lang lashes out at "all sorts of stupid things" in mathematics books, exercises that destroy imagination and make mathematics ugly.

For Lang mathematics is like music. "Mathematics is not just numbers, anymore than music is [just] notes." As one enjoys music, so should one enjoy mathematics: "You can get a high with mathematics," says Lang.

Getting a high with mathematics is the whole point of Lang's lectures and dia-



logues. Some of those who heard him in person undoubtedly did realize a distinct feeling of pleasure at understanding mathematical truth. But reading transcripts is not the same as participating in the dialogue, any more than listening to music is the same as performing it. Like listeners at a concert, readers expect a polished performance.

Unfortunately, these volumes do not provide polish. The combination of transcription and translation (albeit just from French) makes the text idiomatically bumpy. Instead of moving swiftly with the flow of ideas, the reader is constantly interrupted by repetition and unedited circumlocutions that are common to spoken language but distracting in written text. One frequently wishes for tighter editing to convey the same ideas with the same informality of presentation, yet in more compact, lucid prose.

Despite the impediments of style, these volumes offer the excitement of minds engaged in the act of mathematical composition—weaving known themes into new forms, discovering unanticipated harmonies, and forging grand summations in which competing themes converge. They enable an attentive reader to encounter the beauty of mathematics.—*Lynn Arthur Steen, Mathematics, St. Olaf College*

The Encyclopedia of Birds

Christopher M. Perrins and Alex L. A. Middleton, eds. 463 pp. Facts on File Publications, 1985. $35.

The editors have undertaken the ambitious project of producing a book containing a scholarly discussion of every family of birds in the world. Perrins and Middleton have succeeded magnificiently. I know of no other book with such an enormous scope that nevertheless manages to incorporate so much up-to-date detail in such a relatively short space.

One unique feature of this encyclopedia is that it is not arranged alphabetically, but rather, after a short general introduction, is arranged in taxonomic order (one not entirely familiar to North American readers, perhaps, but clearly explained in the introduction). This organization is a major advantage, since all related bird groups are treated together without regard to the first letter of their English names.

Another unique feature is the fine visual summary box that introduces each of the book's taxonomic sections. Each shaded box presents a concise overview of the groups to be treated, complete with a list of the orders and families involved, an estimate of the number of species, excellent range maps color-coded by family, and innovative silhouetted depictions of the birds' sizes as compared to a human. Throughout the book, one frequently encounters boxes containing slightly finer print, presenting a somewhat more in-depth treatment of certain aspects of the birds' biology, e.g., diving in penguins, courtship in antbirds, the effects of pesticides on eggshells (entitled "shell shock"), and the like. This feature makes the book's information still easier to locate and use.

The introductory section entitled "What Is a Bird?" contains one of the clearest and most succinct presentations of functional avian anatomy that I have ever read. While the individual facts are clearly not unique to this book, the clarity of synthesis certainly is.

Few books are entirely without flaws. The illustrations are a mixed bag: many superb color photographs, with paintings ranging in quality from excellent to rather poor. The paintings serve the good purpose of showing a wide variety of birds in a small space. Yet for this they should be to scale (the major advantage of a com-

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