Renewing Undergraduate Mathematics

by Lynn Arthur Steen

Undergraduate mathematics is a major educational conduit for our nation's scientific personnel—not only for future engineers, physicists, and mathematicians, but also for computer scientists, statisticians, school teachers, physicians, economists, and business executives. Some type of undergraduate mathematics is required for virtually every scientific and engineering degree. Undergraduate mathematics is to scientific research what basic research is to applied science, the supplier of intellectual resources.

As science changes, so also must the pattern of undergraduate mathematics. On top of this, mathematics itself is changing dramatically—in content, scope, and application. Powerful and ubiquitous new applications signal to the educated public that mathematics is no longer, if it ever was, the sterile, ethereal, axiomatic exercise of journalistic caricature. The truth is that mathematics is not just being applied, but is being continually created in response to challenges from science, from technology, and from other parts of mathematics itself.

Although new courses such as data analysis, operations research, and discrete mathematics are finding their place in the curriculum, very few courses in the typical undergraduate program give students a realistic sense of the true nature of contemporary mathematics, either pure or applied. Unlike their peers in the natural sciences, undergraduate mathematics students rarely move beyond classroom exercises involving mathematics that is several decades (or even centuries) old. Prevented by curricular constraints from seeing how mathematics is created, students too often view mathematics only as a powerful but static collection of tools to be learned (or worse, memorized) and then applied. They fail to see career options in a field that is presented as a *fait accompli.*

Undergraduate mathematics bears major responsibility for the future well-being of American society. Collegiate mathematics must provide courses for future scientists, programs for prospective elementary and secondary school teachers, remedial courses for those entering college unprepared in mathematics, general education courses for students not majoring in a quantitative discipline, strong majors for those intending to enter graduate school, and a variety of service courses ranging from elementary statistics to advanced operations research. Moreover, in many institutions, mathematicians must also teach computer programming and elementary computer science.

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As mathematics needs to be continually created to provide new tools for science and industry, so the undergraduate curriculum needs to be continually renewed to reflect the changing nature of mathematical practice and scholarship. Yet the limited resources of undergraduate mathematics departments are now thinly spread over an enormous variety of elementary service courses, leaving virtually no time or energy for the in-depth study necessary to renew faculty initiative or to develop innovative programs.

Signs of distress are not hard to find. The number of degrees awarded in mathematics is only about half of what it was ten or fifteen years ago. Enrollments at the elementary level are double what they used to be, and faculty work loads have increased significantly. Demand for computer science is distorting enrollments and depleting the pool of young prospective mathematicians. Dual salary scales are demoralizing faculty at the same time as budgets for library resources and travel are diminishing.

It is time for the mathematical community—researchers, teachers, and users—to join in a common cause to renew undergraduate mathematics. We need to do more than stimulate the curriculum. We must examine and respond to the realities of student interests and preparation. We must articulate standards for the profession that will enhance the morale and effectiveness of college mathematics teachers. Most importantly, we must engage ourselves and our students in the excitement of creative mathematics applied to challenging scientific and societal problems.

Students

Any analysis of undergraduate mathematics must begin with informed knowledge of our students, who arrive in college having studied mathematics in some form for most of their school years.

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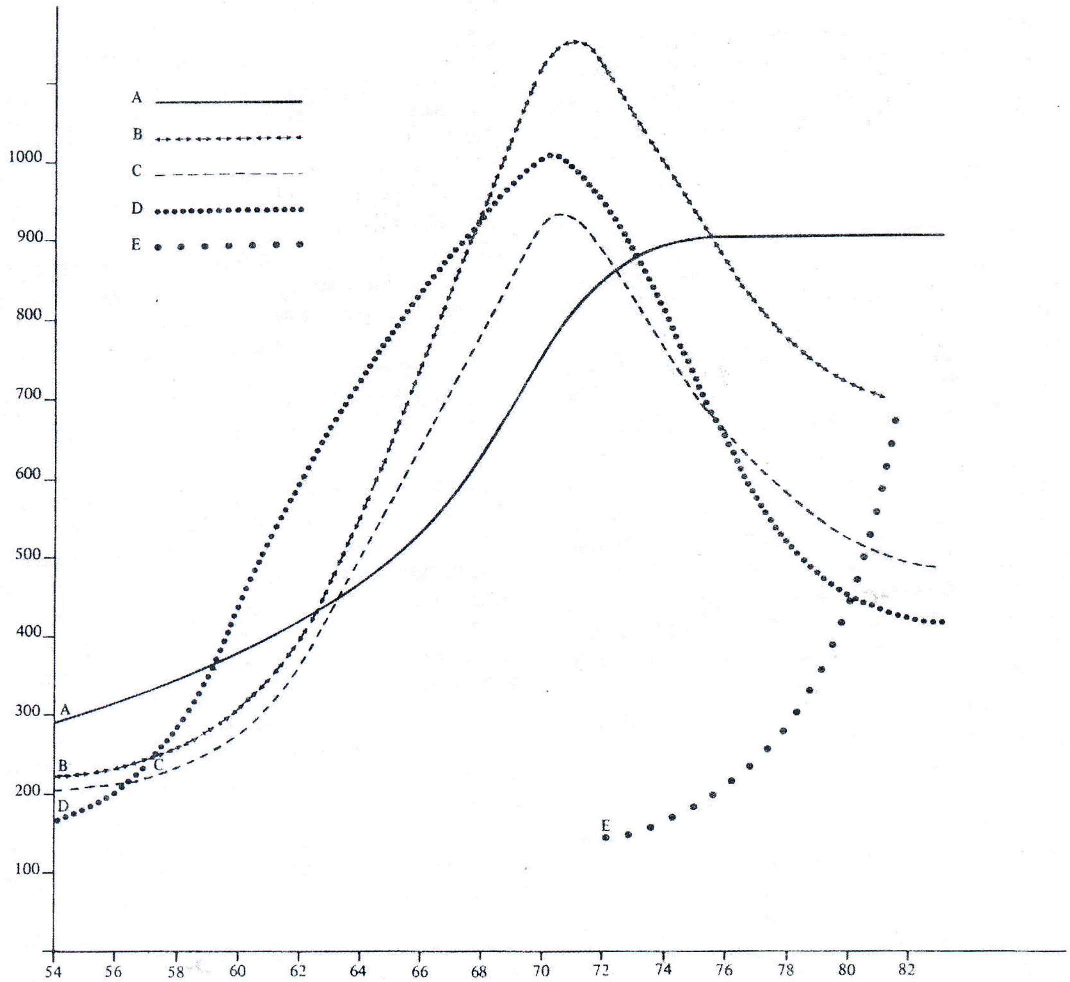

The Rise and Fall of U.S. Mathematics Degrees. This composite graph pictures smoothed data for bachelor's and doctor's degrees for the preceding three decades:

Curve A represents the total number of bachelor's degrees awarded in the United States, in units of 1,000. Following a long period of growth, the number of degrees has stabilized during the last ten years at just over 900,000.

Curve B represents the total number of Ph.D. degrees awarded in the United States in mathematics and statistics. This number has now fallen by 40% from its peak of about 1200 in 1970-1971.

Curve C represents the number of U.S. citizens who received a Ph.D. in mathematics or statistics. As a percent of the data represented by Curve B, it has fallen from about 90% to just under 60%.

Curve D represents the total number of U.S. bachelor's degrees in mathematics, in units of 25. This number also peaked in 1970, at about 25,000; now it has dropped to half that level.

Curve E, also in units of 25, represents the growth of bachelor's degrees in computer and information sciences. This curve crossed Curve D, the bachelor's degrees in mathematics, in 1980.

The effect of this study is to add to the already great natural variance in mathematical aptitude an enormous variance in both competence and attitude. The latter two characteristics frequently overwhelm the former, and the three together produce a diversity in performance that is almost beyond comprehension: the range of mathematical performance of college freshmen spans ten years of education (grades 7–16), but the distribution is skewed decidedly towards the lower half of this range.

There are approximately four million persons in each age cohort among students of school age; that is a bit more than 1/70th of the total U.S. population. Nearly 30% do not graduate from high school. (The percent of students who finish high school has been declining slightly for the past twenty years, after increasing steadily for the past century.) Of the three million persons who do graduate, about half, approximately 1.5 million, enter college.

About two-thirds of those who enter college have not studied precalculus mathematics. This means that each year about one million students enter higher education without having completed the full program of high school mathematics. Half of those have taken a second algebra course, the other half have not. Thus half a million college students—those who never took Algebra II plus many of those who forgot most of what little they may have learned in that course—enter college needing extensive review of elementary algebra, sometimes including what is euphemistically called "arithmetic for college students."

The best-prepared third of college freshmen, another half a million, are well prepared for college mathematics courses. Approximately 20%, or about 100,000, have actually taken calculus. These represent the top 3% of high school seniors, yet only about one-fourth of those learn enough calculus in high school to receive college credit for the course. Results released at the end of 1984 from the Second International Assessment of Mathematics show that U.S. high school students who have taken calculus score barely above the median when compared to the top 10–15% of students from other countries.

Here is a tabular estimate of the mathematics placement of 18 year old students in the United States:

30%	Do not finish high school
35%	Finish high school; do not go to college
12%	Enter college needing elementary algebra
10%	Enter college ready for precalculus
10%	Enter college ready for calculus
2%	Enter college having completed some calculus
1%	Enter college with some calculus credit
100%	

Last year the National Institute of Education released a report called *Involvement in Learning: Realizing the Potential of American Higher Education.* This report highlights two other student characteristics that are crucial to an understanding of the total undergraduate environment: 40% of all students study part-time, and 40% of all students are over the age of 25. Undoubtedly there is a large overlap between these two groups. And since their impact on undergraduate enrollments is weighted by the part-time nature of their study, the overall impact on enrollment patterns is far less than 40%. But as people, as decision makers, and as voters, these older part-time students represent an important fraction of the individuals studying undergraduate mathematics.

Enrollments

Enrollment patterns in mathematics are difficult to find and interpret, partly because the mathematics profession does not spend much of its resources in keeping track of such data. By interpolating among several sources, I made the following estimates of the distribution of enrollments (in thousands) in beginning college mathematics courses:

%	Enrollment	Course
22%	700	Remedial
18%	600	Calculus
18%	600	Programming
16%	500	Precalculus
13%	400	Elementary Statistics
5%	150	Finite Mathematics
3%	100	Computer Science
3%	100	Discrete Mathematics
2%	50	Mathematics Appreciation
100%	3200	Total

(For check-point comparison, American Mathematical Society (AMS) data for 1983 indicates a total fall enrollment in mathematics and statistics courses of 2.4 million. The table above includes computer programming and elementary computer science, which about accounts for most of the extra enrollments. Since computer courses are taught in both mathematics and computer science departments, it is very difficult to account precisely for the impact that undergraduate computer courses have on mathematics enrollments. The estimate above is primarily for those computer courses taught by or in mathematics departments; it does not count computer science enrollments in departments that are totally separate from mathematics.)

In contrast to the total enrollment of about 3 million in elementary courses in the mathematical sciences, the total enrollment in advanced (postcalculus) undergraduate mathematics is only about 200,000, two-thirds in applied analysis (differential equations and related subjects), one-third in other parts of core mathematics. In other words, over 90% of the enrollments in undergraduate mathematical science are in lower division service courses.

Much of this service load is of recent origin, driven by a society convinced that mathematics, in modest doses, is useful, and perhaps even profitable. These pragmatic forces have reshaped the whole undergraduate culture: in the last fifteen years or so, the number of bachelor's degrees awarded in the arts and sciences has declined by about 50%, while the number awarded in job-related fields has more than doubled. Whereas at the end of the 1960s bachelor's graduates roughly were divided equally between those in arts and science and those in specialty programs, now only about 20% of graduates are in the arts and sciences.

These changes have had a significant impact on enrollment patterns in mathematics. Indeed, in the past fifteen years undergraduate mathematics enrollments in the mathematical sciences have increased twice as fast as has the general undergraduate population, but this increase has occurred totally in the elementary part of the curriculum:

	1970	1985	%
Remedial & Precalculus	800	1800	
Calculus	450	600	
Computer Programming	50	600	
Advanced Mathematics	300	200	
Total Mathematical Sciences	1600	3200	100%
Total B.A. Degrees	780	930	20%
Total F.T.E. Undergraduates	6700	9500	42%

Graduates

About two-thirds of the students who enter college actually graduate: each year there are just under one million bachelor's degrees conferred in the United States. Of these, about 1% are in mathematics, that is, about 10,000. (Here mathematics includes statistics, but not computer science.) About twice that number are now receiving bachelor's degrees in computer and information sciences. In 1970, there were about 30,000 bachelor's degrees in the mathematical sciences awarded in the United States—90% (27,000) in mathematics, 10% in computer and information science. Since then mathematics degrees have steadily declined and computer science degrees have steadily increased, with the total staying relatively constant. Now there are about 11,000 bachelor's degrees in mathematics, and over 20,000 in computer science.

As bachelor's degrees in mathematics have declined, so have Ph.D. degrees. But even more important, the percentage of U.S. citizens receiving the Ph.D. in mathematics has also declined, from about 80% to under 60%. This has led to a compound decrease of about 50% in the number of Americans receiving doctoral degrees in mathematics. In fact, the number of degrees to U.S. citizens in core mathematics is now as low as it was in the 1960s. Here is a comparison, in round numbers:

	1962		1972		1982	
Mathematics Ph.D.'s	Total	U.S.	Total	U.S.	Total	U.S.
Core Mathematics	400	370	920	720	510	310
Applied Mathematics	50	40	120	100	120	80
Statistics	50	40	150	110	150	90
Mathematics & Statistics Total	500	450	1190	930	780	480
Computer Science	–	–	170	130	260	160
Mathematical Sciences Total	500	450	1360	1060	1040	640

Faculty

There are currently about 25,000 full time mathematics faculty members in United States institutions of higher education. One fourth are in Ph.D. granting institutions, one half in master's and bachelor's degree institutions, and one fourth in two year colleges. In addition, there are another 20,000 persons who teach mathematics part-time: 9,000 in the two year colleges, 5,000 in the four year institutions, and 6,000, mostly teaching assistants, in the universities.

About two-thirds of all full-time mathematics faculty hold doctoral degrees. In 1965 only 35% of the faculty appointments at the four-year colleges were filled by persons with a doctorate; that percentage has now doubled, partly in response to a national effort in the late 1960s to improve faculty credentials in mathematics. The large load of precalculus instruction coupled with increasing demand for computer science instruction—where Ph.D. degrees are rare—suggests the current percentage of Ph.D. faculty may represent a stable long-term balance of faculty preparation with teaching needs.

	Mathematics Faculty			
	2 Year	4 Year	Ph.D.	Total
Full Time	6,000	12,000	7,000	25,000
Part Time	9,000	5,000	6,000	20,000
F.T.E.	8,000	13,000	9,000	30,000
Ph.D.	1,000	8,500	7,000	16,500
% of F.T.E.	15%	70%	100%	66%
Total Faculty	15,000	17,000	13,000	45,000

For comparison, the U.S. membership in the Mathematical Association of America (MAA) is about 18,000, and in the AMS about 14,000; together, about 25,000 U.S. residents belong to one or the other of these major professional mathematics societies. That is more or less the same as the total full-time mathematics faculty, although of course neither group is quite identical with the other.

The age distribution of the mathematics faculty is very uneven: 20% are older than fifty-five; 50% are between forty and fifty-five; only 30% are younger than forty. Far from being uniform, the age distribution of the mathematics faculty is almost bell-shaped. Just from the Ph.D. faculty alone, there will be 200 retirements per year for the next few years, rising to about 400 per year by the year 2000. Overall, higher education has to replace over 3000 Ph.D. mathematicians during the next fifteen years, about half of what will be produced at present rates.

Recent AMS data shows that only two-thirds of the new doctorates accept first jobs in colleges or universities. The median starting salary for such academic positions is $23,000. The undergraduates that these new Ph.D.'s teach, leaving college with only a bachelor's degree, receive median starting (12 month) salaries of $23,400. The corresponding salary to a new mathematics Ph.D. in industry is about $36,000.

Finally, I note that the "David Report", *Renewing U.S. Mathematics* (National Academy of Sciences, 1984) [reprinted in the *Notices*, August 1984, pages 435–466; October 1984, pages 570–616] documents U.S. plans to introduce several hundred supercomputers during the next decade into academic, industrial and government facilities. Each such machine requires, on average, about a dozen mathematical scientists with sophisticated knowledge of the mathematics of computation. Several hundred such machines will require several thousand new mathematicians. The implications of this demand for the supply of undergraduate faculty are staggering.

Undergraduate Mathematics

The mathematics covered these days in typical undergraduate programs can be divided roughly into three parts: the elementary, the old-fashioned, and the experimental. That may sound unfair, and it may be; but it has a grain of truth. As enrollment data shows, many courses are really not appropriate to collegiate level instruction. Many others frequently contain little or no hint of modern mathematics. And the rest are new courses whose contribution to a unified curriculum is totally untested.

We need courses like these. We need elementary courses to meet students on their own terms; we need traditional courses to convey the classics of mathematics; and we need experimental courses to probe for new areas worthy of undergraduate study. But the nature of contemporary mathematics demands something more.

In an essay titled "Ordering the Universe: The Role of Mathematics," an appendix to the David Report, Arthur Jaffe writes about the enormous breadth of modern mathematics, pure and applied. Here is a sample from his survey:

- Fourier analysis, from fast Fourier transform to pseudodifferential equations;
- Simple groups and number theory, applied to algorithms and computational complexity;
- Numerical mathematics, used for nuclear reactors and computerized tomography;
- Compact groups, used in mathematical physics to represent quarks and supersymmetry;
- Fibre bundles and connections, used for gauge theory in electrodynamics;
- Poincaré conjecture, in four dimensions, yielding exotic Euclidean spaces and explanations of solitons;
- Algebraic geometry, applying the Riemann-Roch theorem to generate error-correcting codes;
- Time series analysis used for seismic exploration for oil;
- Chaotic behavior in dynamical systems, related to the onset of turbulence as well as to the theory of fractals;
- Parallel computation and unbounded memory, suggesting radically new algorithms for numerical mathematics.

This list of current mathematical research topics is neither elementary, nor old-fashioned, nor experimental. It consists of classical mathematics—analysis, algebra, topology—mixed heavily with physics and engineering, employing modern computer tools to model significant scientific phenomena. It shows a vigorous science rooted in the rich soil provided by generations of mathematical giants.

We cannot teach all this mathematics to undergraduates. But we must, somehow, teach the foundations of this mathematics, while at the same time providing glimpses of the structure that this foundation can support. To do that will require a new synthesis of classical and modern topics, not merely the unstructured aggregation of traditional courses with experimental alternatives.

It is not my intent here to discuss the many curricular changes affecting undergraduate mathematics. The enrollment patterns cited above indicate the extent to which the rushing waters of mathematics have moved from a narrow deep gorge to a flat broad plain. Twenty years ago the Committee on the Undergraduate Program in Mathematics (CUPM) helped guide the undergraduate curriculum to a stable consensus on a core of undergraduate mathematics. Today that consensus is shattered: in its most recent statement CUPM reported that there is no longer any consensus on specific advanced subjects that should constitute the core of undergraduate mathematics.

Mathematics is not the only discipline suffering from a dissolution of consensus on purpose and direction. A recent report by the Association of American Colleges (AAC) called *Integrity in the College Classroom* decries what it calls the "decline and devaluation" of undergraduate education: "A consequence of the dispersal of authority over the curriculum is... unhappy disarray, the loss of integrity in the bachelor's degree." The AAC report cites the narrow graduate-school professionalism of faculties as the root cause of the identity crisis in undergraduate teaching, and seeks "to revive the responsibility of the faculty as a whole for the curriculum as a whole."

Mathematics is a party to the decline of undergraduate education, sharing both in responsibility for the decline as well as in its consequences. What used to be a focused albeit narrow curriculum is now too often a smorgasbord of unrelated courses. As demand for new applications proliferate, the focus of the undergraduate curriculum disintegrates. In many departments mathematics faculty now devote more teaching effort to computer programming than to calculus.

This curricular change is a two-edged sword. While it has diminished the strength of traditional core mathematics—what most of us were trained in during our graduate studies—it has at the same time multiplied the linkages between mathematics and other disciplines. No longer are the concepts of mathematics only used in physics and engineering. Now they can be found in linguistics, medicine, psychology, agriculture, music—virtually every subject taught in an undergraduate curriculum. The connections between mathematics and other subjects are often mediated by computer science, but real mathematics lurks immediately beneath the surface. Although most of us do not yet realize it, and many may not even welcome it, the mathematics faculty has within its discipline a legitimate responsibility for linkages to the whole undergraduate curriculum. Mathematics as a discipline is uniquely positioned to help play a major role in the renewal of undergraduate education.

Teaching and Research

The renewal of collegiate mathematics will require imaginative effort in curricular reform, both within the mathematics major and in various interdisciplinary programs. It will require exciting new approaches that attract the best young minds of the next generation, as well as a continual struggle to encourage good students to pursue graduate work in the mathematical sciences. But most of all, it will require sound and productive programs of faculty evaluation and faculty development for those 25,000 members of our current mathematics faculties.

In every field, the vitality of undergraduate education depends on effective links between teaching and research. Such links are especially important in mathematics, because the field is changing so rapidly. They are also especially difficult to form, since the frontier of mathematical research is so remote from the reality of undergraduate courses. The links between teaching and research in mathematics are long, fragile, and easily broken. Especially for this reason, the relation between teaching and research is an important and crucial aspect of faculty renewal and faculty evaluation.

Renewing U.S. Mathematics calls for vast increases in support for mathematics research, especially in the leading Ph.D. granting institutions: "The health of the mathematical enterprise in the United States hinges on the strength and vitality of the departments in the leading research universities." This report also contains a careful analysis of research productivity in the mathematical sciences, cross-checked in several different ways. It concludes that the number of productive research mathematicians is about 3,000, including 2,600 established and 400 young investigators. One measure of "productivity" was three papers in five years that were reviewed in *Mathematical Reviews*; another was peer review that judged their work equivalent to that already being supported through National Science Foundation (NSF) and Department of Defense (DOD) research grants. It is clear from this study that only a small minority—about 10% of the total, or 20% of the Ph.D. holders—of U.S. mathematics faculty are productive researchers according to these criteria. This observation has important implications for faculty renewal and faculty evaluation.

Every college and university sets standards of professional work for permanent members of its faculty. Research universities usually have three distinct missions: teaching, research, and service; faculty responsibilities at these institutions parallel the mission of the institution, requiring significant contributions in each area for its own sake.

The majority of post-secondary institutions, however, define teaching as their primary mission. Yet even most of these institutions, at least all the four year institutions, require significant professional activity of faculty to insure that they remain intellectually alive and actively in contact with their discipline. The vast majority of faculty at these schools engage in research and professional activity not so much to advance the frontiers of research as to maintain their vitality as teachers and to provide, by example and by experience, a context in which their students can taste the excitement of creative mathematics. It is this aspect of professional work that is especially important in mathematics, yet too often overlooked in faculty tenure and promotion reviews.

The relation of teaching to research in mathematics is crucially important and virtually unique among undergraduate disciplines. Professional activity is enormously important in mathematics because of the rapid growth of the mathematical sciences. Teaching that is divorced from professional activity may be effective and popular, but it cannot long remain intellectually honest. The only way for a curriculum in the mathematical sciences to remain current is for the faculty to remain professionally active.

For too long mathematics and mathematics teachers have suffered from a rigid, narrow definition of professional activity. To save face with our peers in the sciences and humanities, we expect of ourselves a productive research program; to save face with our peers in mathematics, we adopt the mathematician's elite definition of research. The result too often is confusion, frustration, and well-intentioned hypocrisy in faculty tenure and promotion proceedings.

Morris Kline argued forcefully in his provocative 1977 book, *Why the Professor Can't Teach,* that mathematics must re-establish respect for scholarship, for research in its traditional meaning. In this view, a teacher's time and energy should be devoted both to instruction and to that kind of scholarship which is the complementary aspect of good teaching. The breadth of the mathematical sciences, the importance of renewed links between teaching and research, the rapid creation of new mathematics, as well as the David report's conclusion that only 10% of college mathematicians are productive researchers—these and other signs suggest that it is time to establish a new definition of professional work for college mathematicians.

Professional Work in Mathematics

Professional work in mathematics, as in any field, must be public—that's the root of the word "publication." But it need not be restricted to narrow, traditional research publications. It should embrace all published works (texts, research papers, reviews, expository articles, classroom notes), presentations at meetings and at other institutions, leadership in professional organizations, arranging professional workshops, and consulting for government, industry or academic institutions. The important common element is the scrutiny and review afforded by public presentation: this is vital to both the individual and the institution as an external measure of the significance of the work. Moreover, public presentation imposes on the individual a healthy discipline in organizing ideas and thinking systematically about key issues in the mathematical sciences.

The creation of new mathematics expresses as nothing else can the fundamental processes of mathematics, and an active research program in a department can help stimulate not only new ideas but also new modes of thought. But it is not something we can demand as a *sine qua non* for either promotion or tenure. It is, rather, one option among many.

To balance my appeal to authority, I add supporting evidence from Peter Hilton, who once wrote a scathing review of Kline's book. But on this issue they seem to agree: "I believe," writes Hilton, "that promotion and tenure should be the reward of outstanding work of an imaginative and innovative nature. Such outstanding work could be in the field of mathematical research, but does not have to be. Thus it is perfectly possible and, today, more important than ever, to show imagination, energy and enterprise in the development of new courses and the modernization of old ones."

Directly or indirectly, all professional activity relates to teaching. Teachers who are active imbue their courses with a spirit of current thought. Yet only rarely in mathematics will the content of significant research translate into material suitable for undergraduate instruction. It is in this respect that mathematics differs from most other fields. A Shakespeare scholar can relate current research to undergraduate courses, as can a biochemist studying techniques of recombinant DNA. But the mathematician working on shock waves or gauge fields cannot readily relate this work to any typical course in the undergraduate curriculum. However, the process of mathematics is continually renewed by professional activity, and it is the process more than the content that matters in effective teaching.

The gulf between undergraduate instruction and mathematical research is much easier to span in the newer applied subjects than in traditional core mathematics. These subjects appeal to students not only because they are new and applicable, but also because they are near the frontier. Undergraduates need to experience the euphoria of discovery in order to taste the true nature of mathematics. The ability of instructors to lead students to the brink of unsolved problems in these newer areas is in itself a substantial reason to emphasize these topics in the undergraduate curriculum. Moreover, what is good for students is also good for the faculty: interdisciplinary work applying mathematics to problems in other fields provides a marvelous opportunity for college mathematics teachers to become professionally active.

Typically, the links between scholarship and teaching that emerge in mathematics relate to development of new courses or entire new curriculum structures, to the integration of computing and applied techniques into traditional mathematics, to supervision of independent study in areas that reach into unfamiliar territory, to development of innovative curriculum materials for new courses, or to development of computer software and documentation. In cases such as these, professional work is often focused on local issues and, for this reason, may not lead to significant public exposure. It is, nevertheless, important for the department and for the individual.

We, the mathematical community, must work to establish effective mechanisms to evaluate and reward professional activity in the context of each institution's special mission and objectives. Evaluation must recognize the varied purposes of research and professional activity. Some research—the minority—benefits mathematics directly by advancing the frontiers of knowledge. Most research and professional activity benefits mathematics indirectly by invigorating the faculty, stimulating students, and refreshing the curriculum. Both are necessary for mathematics to thrive, and both must be recognized and suitably rewarded.

An Agenda for Renewal

Successful undergraduate mathematics requires a faculty that is active, scholarly, and vigorous. To revitalize undergraduate mathematics we must infuse the undergraduate years with the spirit if not the details of contemporary mathematical activity. We must support exemplary programs that encourage students to major in mathematics. We must encourage creativity in developing programs for prospective school teachers as well as for prospective scientists. We must reward those who provide effective courses in "mathematical literacy" for future lawyers, politicians, and citizens. And perhaps most important, we must establish standards for faculty evaluation that promote innovation in teaching and scholarship in mathematics.

To be specific, an agenda for renewal should include such things as:

- Undergraduate scholarly activities, to provide for mathematics students what the laboratory does for science students and the stage for drama students.
- Curriculum modernization, to bring into the undergraduate curriculum the most exciting ideas of modern applied mathematics.
- Interdisciplinary efforts, to show how mathematical ideas can illuminate many of the broad issues—energy policy, economic theory, strategic doctrine—that capture the imagination of undergraduate students.
- Redefinition of the core of undergraduate mathematics, to determine what subjects all mathematics majors should know.
- Recognition of scholarship rather than narrow research as the true mark of professional activity for college mathematics teachers.

For the rest of this decade mathematics departments will continue under great stress. We live in the shadow of computer science, the glamor stock of academia. In contrast to computing, mathematics appears as a cerebral abstraction, isolated from reality.

Industry is hiring mathematics graduates as never before; society is pressuring the schools to stress mathematics; and the scientific research community has endorsed mathematics as one of the priority areas for support in years ahead. It is important that we not let these opportunities slip away.

Teachers of undergraduate mathematics must make every effort to convey not only to our students but also to our colleagues and to the general public the contributions mathematics is making to society. The great lesson of the past twenty years is that the most abstract ideas are the most powerful, and the most abstract thinkers the most versatile.