Crisis and Renewal in College Mathematics

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Undergraduate mathematics is a major conduit to careers in science and in business. Some college mathematics or statistics is required for virtually every professional-degree program. Now mathematics is changing dramatically—in content, in scope, and in application. Powerful and ubiquitous examples of new applications are signaling to the educated public that mathematics is no longer—if it ever was—a sterile, arcane exercise. And it is not just being applied in new ways; it is being continually created.

As mathematics needs to be continually created to provide new tools fro science and industry, so the undergraduate curriculum needs to be continually renewed to reflect the changing nature of mathematical practice and scholarship. Yet the limited resources of undergraduate mathematics departments are now thinly spread over an enormous variety of elementary courses.

Those departments must provide courses for future scientist, programs for prospective teachers in elementary and secondary schools, remedial work for students who come to college unprepared in mathematics, general courses for students not majoring in a quantitative discipline, strong major programs for students intending to enter graduate school, and a variety of service courses ranging from elementary statistics to advanced operations research. Moreover, at most institutions, mathematicians must also teach computer programming and elementary computer science. The strain of meeting these diverse obligations leaves virtually no time or energy for the thorough study necessary to renew faculty initiative or to develop innovative programs.

Reasons for the strain are not hard to find. Nearly two-thirds of college freshmen are not prepared to study college mathematics, and must instead take or retake some parts of high-school mathematics. During the last 15 years, total undergraduate enrolment in mathematics has doubled, but enrollment in advanced courses has been cut in half. Now less than 10 per cent of undergraduate enrollment is in post-calculus courses.

Bachelor's, master's, and doctoral degrees in mathematics have declined by over 50 per cent. Fewer than 60 per cent of the Ph.D.'s granted by American universities in the mathematical sciences go to U.S. citizens. Projected faculty retirements in excess of 300 per ear, coupled with vigorous demand for mathematicians from the supercomputing industry, virtually insure that the country will face a serious shortage of mathematicians just as demand for mathematical knowledge and skills reaches record levels.

A 1984 report by the National academy of Sciences, *Renewing U.S. Mathematics*, cites case after case of mathematics' being pressed into service by modern science: compact groups in mathematical physics, algebraic geometry in error-correcting codes, numerical methods for computerized topography—to name just three examples. These examples show classical mathematics—analysis, algebra, topology—mixed heavily with physics and engineering, employing modern computer tools to model significant scientific phenomena. They show a vigorous science rooted in the rich soil provided by the work of generations of mathematical giants.

We cannot teach all this mathematics to undergraduates. But we must, somehow, teach its foundations, while at the same time providing glimpses of the structure that the foundations support. To do so will

require a new synthesis of classical and modern topics, not merely and unstructured aggregation of traditional courses with experimental alternatives.

The curriculum must be changed to multiply the links between mathematics and other disciplines. No longer are the concepts of mathematics used mainly in physics and engineering. Now they can be found in linguistics, medicine, psychology, agriculture, music—virtually every subject in the undergraduate curriculum. The connections between mathematics and other subjects are often mediated by computer science, but real mathematics always lurks immediately beneath the surface.

The renewal of college mathematics will require imaginative effort in curricular reform, both in the mathematics major and in various interdisciplinary programs. It will require exciting new approaches to attract the best young minds, as well as a continual struggle to encourage good students to pursue graduate work in mathematics. But most of all, it will require sound and productive programs of faculty evaluation and faculty development.

In every field, the vitality of undergraduate education depends on effective links between teaching and research. Such links are especially important in mathematics, because the field in changing so rapidly. They are also especially difficult to form, since the frontier of mathematical research is so remote from undergraduate courses. The links between teaching and research in mathematics are long, fragile, and easily broken.

According to *Renewing U.S. Mathematics*, the number of productive research mathematicians in the United States is about 3,000, or 10 per cent of the total mathematics faculty members. The majority of mathematics faculty members are employed at institutions whose primary mission is teaching. Yet most of those institutions, certainly all the four-year colleges, require significant professional activity of their faculty members to insure that they remain intellectually alive and in touch with their disciplines.

Faculty members at these institutions, the vast majority of them, engage in research and professional activity not so much to advance the frontiers of research as to maintain their vitality as teachers and to provide, by example, ad context in which their students can experience the excitement of creative mathematics. It is this aspect of professional work that is especially important in mathematics, yet too often overlooked in tenure and promotion reviews.

Because of the rapid growth of the mathematical sciences, the relation of teaching to research is crucially important and virtually unique among undergraduate disciplines. Teaching that is divorced form research may be effective and popular, but it cannot remain intellectually honest. The only way for a mathematics curriculum to stay current is for the faculty to be professionally active.

For too long, mathematics and mathematics teachers have suffered form a rigid, narrow definition of professional activity. To save face with our peers in the other sciences and the humanities, we demand of ourselves a productive research program; to save face with our peers in mathematics, we adopt the mathematician's elite definition of research. The result is often confusion, frustration, and well-intentioned hypocrisy in tenure and promotion reviews.

The breath of the mathematical scienc3es, the importance of inks between teaching and research, the rapid creation of new mathematics, as well as the fact that only 10 per cent of college mathematicians are productive researchers, point to the need to establish a new definition of professional work for mathematics faculty members.

Professional work in mathematics, as in any field, must be public—that's the rood of the word "publication"—but it need not be restricted to narrow, traditional research publications. It should embrace all published work (including reviews and exposition), presentations at meetings, leadership in professional organizations, arranging professional workshops, and consulting. The important common element is the scrutiny and review afforded by public presentation; this is vital to both the individual and the institution as an external measure of the significance of the work. Moreover, public presentation

imposes on the individual a healthy discipline in organizing ideas and thinking systematically about key issues.

The creation of new mathematics expresses, as nothing else can, the fundamental processes of mathematics, and an active research program in a department can help stimulate not only new ideas but also new modes of thought. But it is not something we can demand as a prerequisite for promotion or tenure; it is, rather, one option among many.

Teachers who are active professionally imbue their courses with the spirit of current thought. Yet only rarely in mathematics will the content of significant research translate into material suitable for undergraduate instruction. It is in this respect that mathematics differs from most other fields. A Shakespeare scholar can relate current research to undergraduate courses, as can a biochemist studying techniques of recombinant DNA. But the mathematician working on shock waves or gauge fields cannot readily relate that work to any typical undergraduate mathematics course.

The links that emerge between scholarship and teaching in mathematics usually relate to the development of new courses or entirely new curriculum structures, to the integration of computing and applied techniques into traditional mathematics, to supervision of independent study in areas that reach into unfamiliar territory, to the development of innovative course material, to the development of computer software and documentation, or to investigations of mathematical models in interdisciplinary settings.

A successful undergraduate mathematics program requires a faculty that is active, scholarly, and vigorous. To revitalize undergraduate mathematics we must infuse it with the spirit if not details of contemporary scholarships. We must support exemplary programs that will inspire students to major in mathematics. We must encourage creativity in developing programs for prospective schoolteachers as well as for prospective scientists. We must reward those who provide effective courses in "mathematical literacy" for future lawyers, politicians, and citizens.

For the rest of this decade, mathematics departments will continue to be under great strain. We live in the shadow of computer science, the glamour stock of academe. In contrast to *understanding ?* mathematics appears to be a cerebral abstraction isolated from reality. Yet industry is hiring mathematics graduates as never before; society is pressuring schools to stress mathematics; and the scientific-research community has endorsed mathematics as a priority area for support in the years ahead. The great lesson the of the last 20 years is that the most abstract ideas are the most powerful, and the most abstract thinkers the most versatile.

Chairmen of college mathematics departments must work with administrators to establish effective mechanisms to evaluate and reward professional activity that does not necessarily fit the narrow research tradition. Evaluation must recognize the varied purposes of research and professional activity. Some research—the minority—benefits mathematics directly by advancing the frontiers of knowledge in the field. Most research and professional activity benefit mathematics indirectly, by invigorating the faculty, stimulating students, and refreshing the curriculum. Both types are necessary for mathematics to thrive, and both must be recognized and suitably rewarded.

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