CATASTROPHE THEORY: THE FIRST DECADE

This elegant theory for explaining sudden discontinuities from continuously changing causes is still provoking great excitement and controversy

BY LYNN ARTHUR STEEN

It is now just 10 years since the French mathematician René Thom began relating geometry to biology by means of a new method he felicitously termed "catastrophe theory." Several hundred research papers have appeared since then, some providing refined mathematical analyses of the geometry of the various catastrophes, and others offering tentative applications in fields such as political science, economics, biology, physics. The theory has been hailed by some as comparable only to Newton's Principia, and deprecated by others as merely a routine extension of standard techniques, devoid of special significance.

Catastrophe theory is a device for explaining how discontinuities can arise as the result of continuously changing causes. The collapse of a bridge or the buckling of a beam; the sudden firing of a nerve cell or the differentiation of tissue in an embryo; the sudden loss of self-control, or the eruption of a nervous disorder; the collapse of a stock market or the eruption of riot: all are examples of discrete, discontinuous change resulting from gradually evolving environmental factors. Thom's theory, vastly enriched by empirical and theoretical studies of E. Christopher Zeeman of the University of Warwick, has provided models that "explain" these and related phenomena (SN: 9/14/74, p. 166).

Models employing catastrophe theory frequently begin, as do most mathematical models, with a system of equations whose solutions provide families of smooth paths (called trajectories) that determine, at least in principle, the nature of the system at any time in the future. In smoothly varying phenomena the trajectories will vary smoothly: A small change in the original equations, caused perhaps by changing conditions of the environment or by experimental error, will produce only a small change in the trajectory. But in discontinuous phenomena, a slight change in the initial conditions can produce an enormous change in the qualitative nature of the trajectory: What formerly remained stable might now explode; what formerly cycled in repeated patterns might now become extinct.

When this happens in nature, the corresponding mathematical models develop a divergence (or discontinuity) which traditional methods can handle, if at all, only by *ad hoc* methods. Indeed, the failure of standard mathematical models is so complete that in some cases (for example, in the roll of dice or in the development of an embryo) fully determinate structures yield essentially indeterminate results. Catastrophe theory helps explain how the slightest alteration in the initial state of a system can often have totally unpredictable consequences. It is an important step, says Zeeman, in making the inexact sciences exact.

The roots of catastrophe theory penetrate deeply into the geometric structure of abstract space: geometry governs the possible ways in which solution trajectories may behave. About 10 years ago, Thom showed that the general types of stable solutions that exhibit discontinuities are strictly limited in number. If a system is governed by four or fewer parameters (for example, the four dimensions of space-time) then there are only seven possibilities. These seven so-called elementary catastrophes serve as paradigms for the stable ways in which continuous causes occurring in space-time can produce discontinuous consequences.

Zeeman's example of a catastrophe model for aggression in dogs is a good example of the interaction between theory and fact, between model and reality. Zeeman's work is motivated by psychologist Konrad Lorenz's study of fear and rage in dogs. Experimenters can control the dog's behavior by varying certain stimuli that induce, respectively, rage or fear. When they do this, they discover that curious things happen.

When both rage and fear are at rather high levels the dogs often switch from rage to fear (or vice versa) suddenly and without warning, and are hardly ever observed in a neutral attitude. Moreover, the moment at which attack or flight occurs seems not to be uniquely determined by the values of the fear and rage variables. The same values will produce different results in a manner that appears, at least superficially, to be rather arbitrary.

Zeeman uses the cusp catastrophe—the only nontrivial one of the seven elementary types that can be fully represented by a drawing on a piece of paper—to provide a model for the behavior of the dogs. It does a pretty good job. By proper reading of the model you can see why the sudden reversals in behavior occur, and you can see why there is such a qualitative difference between behavior at high levels of excitement and at lower levels. You can even predict (and confirm experimentally) a divergence effect that occurs when both fear and rage are increased simultaneously. Sometimes the trajectory will move to the top (aggression) sheet of the catastrophe surface, and sometimes it will move to the lower (flight) sheet; subsequent behavior will depend enormously on which of these trajectories was followed, but the experimenter will not know (nor can he predict) which it was.

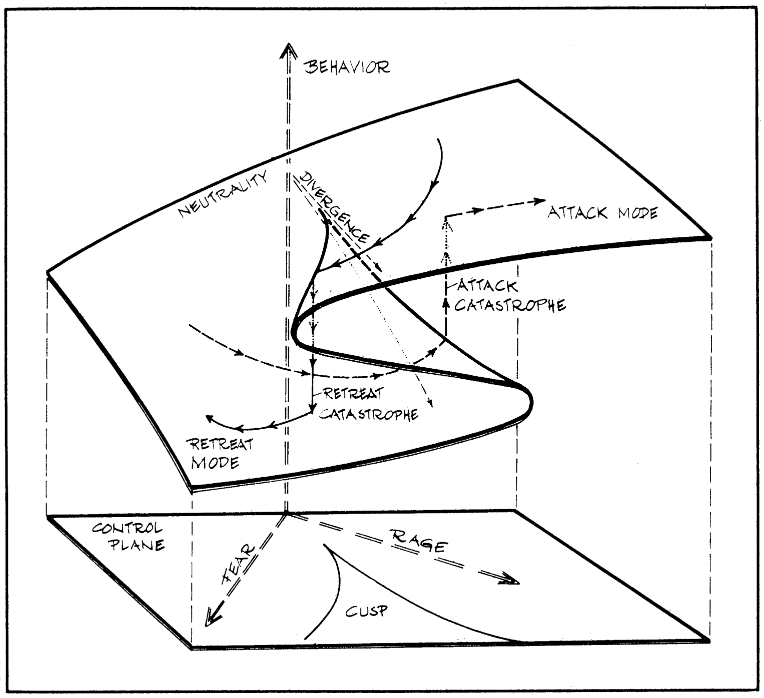
Finally, we can see from the catastrophe surface the bimodal characteristic of the aggression experiments: Excited animals are far more likely to be either very frightened or very angry than to be passively neutral. (Zeeman has an apt anthropomorphic extrapolation of this observation: you can't expect rational [i.e., neutral] behavior from angry people until their level of anger has subsided.)

The Zeeman analysis is very tidy but, at first glance, it may appear to be a bit ingenuous. It does not explain where the cusp surface came from; surely a fertile imagination could invent some other surface that would also explain the same observed facts. Nor does it explain why the animal behaves as it does; surely no one believes that its mind contains a cusp surface which controls behavior.

A serious mathematical model for a scientific phenomenon must be more than a close analogy; it must be a representation of reality that codes in the abstraction of language the connection between first principles and observed behavior. Advocates of catastrophe theory claim that Thom's theorem (on the seven elementary catastrophes) lifts the Zeeman aggression analysis, for instance, from analogy to model. The cusp catastrophe is, they claim, not just a visual representation of observed behavior, but a symbolic representation of the mechanisms that control that behavior.

Stimuli are translated into behavior by electromagnetic wave activity in the brain. The dog's brain is, for all practical purposes, a black box which converts certain gradually changing electrical input signals into output (motor command) signals that occasionally exhibit abrupt discontinuity. We do not know—now nor can we ever expect to know—the exact equations that govern the transformation of incoming to outgoing electrical impulses of the brain. What catastrophe theory does is to describe in a general way how it is possible for continuous signals to be transformed into discontinuous ones.

The electrical activity in the brain (because it is determined by electromagnetic waves) exhibits certain regions known as attractors which determine mood and behavior, in much the same way as a mag-



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Catastrophe model of behavior of dogs stimulated by fear and rage. Possible trajectories illustrate sudden attack, sudden flight and the unpredictability stemming from simultaneous increase in both fear and rage.

netic field will influence the direction of iron filings. As the experimenter excites the dog (in order, for instance, to increase its rage) the external stimuli to the brain will change. The electromagnetic field in the brain changes in response to these impinging electrical signals, and as it changes both the strength and location of the attractors will change. This changing field, called the dynamic, controls the behavior of the animal.

As an attractor moves or strengthens, the dog's attitude changes. It becomes progressively more angry or more frightened. If an attractor diminishes and ultimately disappears, then the attitude of the animal will move rather suddenly—under the influence of electromagnetic forces in its brain—to some new attractor, much as a marble on a rubber sheet will roll down to the bottom of a nearby hole on its trajectory if the depression it was resting in suddenly disappears. The animal's behavior will, in these circumstances, rapidly shift to an entirely different posture—as when it suddenly attacks even while crouched in fear.

The attractors of the dynamic occur at those points where the energy function that controls electrochemical processes of the brain is at a minimum, and these minima are recognized (in calculus) by the presence of a zero derivative. So the possible stable attitudes of the brain are those that correspond to zero derivatives of the potential function. This is precisely what the cusp surface in Zeeman's model represents—points where the potential energy function has a zero derivative.

The mechanism that connects the control variables (the levels of rage and fear set by the experimenters) to the dog's behavior is the changing nature and intensity of attractors in the brain; each point on the cusp surface, because it represents a zero of a potential energy function, represents a possible attractor. Over some parts of the control plane there is only one attractor, while over other parts (inside the cusp) there are two. The transition from one to two attractors (or vice versa) sometimes produces a catastrophic change in the animal's behavior. It doesn't always do this, however; the occurrence of the catastrophic jump depends on which attractor is controlling the behavior at the moment the transition occurs. The cusp catastrophe therefore explains the sudden changes in the dog's behavior as a fully predictable consequence of what the mathematician calls the singularities of the catastrophe map, that is, of the abrupt changes in the nature of the surface representing the attractors of the brain.

Thom's fundamental theorem provides seven elementary ways in which such singularities can develop if the system is governed by not more than four parameters. And, more important, Thom showed that there are no other possibilities that are sufficiently stable to be observed. So even without knowing exact details of the potential function, or of the equations involved in the electrochemical dynamics of the brain, we can be reasonably certain that the mechanism that controls the dog's behavior in the aggression experiments is one of these seven basic forms. Of the seven, the one that fits the details best—with, for instance, two control variables and one behavior variable—is the cusp catastrophe.

Applications of catastrophe theory range from rigorous models to speculative suggestions. The most conservative applications are those that describe physical situations (for example, the buckling of a beam, or the behavior of various "catastrophe machines") in which the governing mechanisms are sufficiently well understood to ensure that the behavior is determined by an appropriate dynamic. It is not uncomon in these situations to be able to write out (and perhaps even solve) the equations that determine the system's behavior. Whether or not this is actually possible, the physical connection between cause and consequence conforms exactly to the hypotheses of the theory which is being applied.

Biological applications, especially to cell growth and embryology, form another major area of application. This is the original area to which Thom applied his theory. He was concerned more with the changing form of things, morphogenesis, than with the calculation of mechanical or physical catastrophes. The logical basis for biological models is similar to that for psychological models: It is the presumption of an ultimate electrochemical mechanism that determines cell or organism behavior. The connection between theory and application in this area is much more tenuous than in physical applications because no one fully understands the biological mechanisms that control cell growth (or, even more, of thought).

A third area of applications, including those "soft" applications mentioned most frequently in popular accounts of catastrophe theory, comprises behavioral and social phenomena in which the catastrophe model explains the behavior but not the mechanism of behavior. Examples such as the collapse of the stock market may (or may not) fit the pattern of one of the elementary catastrophes. But whether they do or not, their behavior is certainly not controlled by the type of dynamic for which Thom's theorem was designed. If soft catastrophe models are useful, it is because they have suggestive power that, in the right context, can "explain" complex events.

A fourth level of application, dominated largely by the fertile writings of René Thom himself, is what might best be termed speculative and suggestive. In his exciting but enigmatic book *Structural Stability and Morphogenesis* Thom uses catastrophe theory as a rich allegory for issues as wide ranging as the structure of society, the development of language and the nature of human activity. The underlying motivation of this imaginative *tour de force* is that succession of form is central to all scientific inquiry. The history of the universe is "a ceaseless creation, evolution and destruction of form, and the purpose of science is to foresee this change of form and, if possible, to explain it."

Geometry is the natural idiom for the study of form. While to Zeeman and many of his colleagues catastrophe theory is an important new device in the tool kit of the applied mathematician, to Thom it is an example of a new paradigm for imposing intellectual order on the universe.

Geometry, argues Thom, is uniquely

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suited to the contemporary challenge of providing abstract models for biological and behavioral phenomena. That part of geometry to which catastrophe theory belongs, called topology, provided mechanisms for inferring global and long-range properties from local, short-range structure. It is a synthetic enterprise that capitalizes on intuition and perception as much as on deduction and inference. The geometric nature of catastrophe theory is worth a thousand abstract symbols to those who build models rather than prove theorems.

Not everyone who has worked with catastrophe theory, however, thinks that it has lived up to its claims. Hector J. Sussman of Rutgers University observes, for instance, that Thom's theorem is a description of all possible ways in which discontinuities can arise in equilibrium surfaces rather than an assertion of certain specific ways in which they must occur in special situations. Thus, he argues, catastrophe models cannot be empirically verified. Conclusions drawn from catastrophe theory models are, Sussman claims, frequently wrong, and if not wrong, then tautological. (A research news article on criticisms of catastrophe theory by Sussman and others is appearing in the April 15 SCIENCE.)

Amazingly, Sussman's views are not too different from those of Thom: "Many people, eager to find for catastrophe theory an experimental confirmation, may embark into precarious quantitative modeling where explicit observable interpretations are given. . . . Needless to say, many (if not all) of these interpretations will break down." Whereas Zeeman and his colleagues have stressed the need for empirical verification, much of Thom's work borders on speculation, being suggestive rather than precise, informal rather than rigorous. Speculation is, according to Thom, "the virtual catastrophe in which knowledge is initiated."

Catastrophe theory is one of the most controversial elements in the contemporary world of higher mathematics. As a mathematical theory, it is above reproach. It is powerful, elegant and deep. Moreover, it provides interesting and useful models for the scientist to explore in his attempts at understanding nature. It does not, however, predict how nature must behave in the same way that, for instance, Newton's laws stipulate the orbits of the planets. "In no case," argues Thom, "has mathematics any right to dictate anything to reality." The most catastrophe theory seems capable of doing at this time is to tell the scientists what forms ("empirical morphologies") he has a right to expect. □

Readers interested in an up-to-date bibliography on catastrophe theory and its applications can obtain one by writing to Dr. Lynn Steen, Department of Mathematics, St. Olaf College, Northfield, Minn. 55057.

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CURRENT MEDICAL DIAGNOSIS AND TREATMENT 1977—Marcus A. Krupp, M.D. and Milton J. Chatton, M.D.—Lange Medical Pubns., 1977, 16th rev. ed., 1,066 p., illus., tables, paper, $16. Intended to serve the practicing physician as a useful desk reference on widely accepted technics currently available for diagnosis and treatment.

EATING DANGEROUSLY: The Hazards of Allergies—Richard Mackarness, M.D.—HarBraceJ, 1977, 180 p., tables, $6.95. Discusses symptoms and causes of allergic reactions, and how they can be alleviated by identifying and eliminating culprit foods from our diets.

LOVEJOY'S COLLEGE GUIDE—Clarence E. Lovejoy—S&S, 1976, 13th ed., 423 p., paper, $5.95. Revised and expanded reference, covers more than 3,600 American colleges and universities.

MEMORY AND MIND—Norman Malcolm—Cornell U Pr, 1977, 277 p., $12.50. Philosopher critically examines accounts of the mental mechanisms of memory, as well as the theories which attempt to explain the phenomena of memory by neural processes in the brain.

THE METRIC SYSTEM MADE SIMPLE—Albert F. Kemp and Thomas J. Richards—Doubleday, 1977, 150 p., illus., $2.50. Explanations are followed by exercises, glossary, conversion and equivalents tables.

NATIONAL FACULTY DIRECTORY, 1977—Gale, 1977, 7th ed., 2407 p., 2 vols., $98. Alphabetical listing, with addresses, of about 449,000 faculty members at nearly 3,000 universities and colleges in the U.S. and Canada.

NEW GUIDE TO THE MOON—Patrick Moore—Norton, 1977, 320 p., 16 plates, drawings, $10.95. A lively description of the moon, its geography, geology and movements, enriched by the recent discoveries. For readers who want to travel there by telescope, spaceship or imagination.

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PEOPLES OF THE SEA—Immanuel Velikovsky—Doubleday, 1977, 280 p., 16 p., 16 plates, maps, $10. Concluding volume of the author's controversial reconstruction of ancient history. This speculative discussion moves the 20th and 21st Egyptian dynasties to the age of the Persian domination over Egypt, a displacement of some eight centuries from conventional chronology.

SCIENCE POLICIES FOR THE DECADE AHEAD: The Second Franklin Conference, Oct. 28-29, 1975—H. Guyford Stever, Chmn.—Franklin Inst. Pr, 1976, 117 p., paper, $6.95. The conference discussed the national, international and humanistic implications of U.S. science policies.

TAKE CHARGE: A Personal Guide to Behavior Modification—William H. Redd and William Sleator—Random, 1977, 183 p., $7.95. Written to help the general public understand the application of this controversial new behavior theory to everyday practical problems.

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VIOLENCE IN ANIMAL AND HUMAN SOCIETIES—Arthur G. Neal, Ed.—Nelson-Hall, 1976, 229 p., tables, $11. Reflecting the magnitude of the problem of destructive violence, the major concern of this interdisciplinary work is violence which is the outcome of the functioning of social systems on any level.

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WHO NAMED THE DAISY? WHO NAMED THE ROSE? A Roving Dictionary of North American Wildflowers—Mary Durant—Dodd, 1977, 224 p., drawings by Eleanor Cooney, $7.95. Handsomely presented compendium of well-researched factual and literary information, traces the origins of the names of more than 100 common wildflowers.