CHALLENGES & TRANSITIONS: UNDERGRADUATE MATHEMATICS, 1990-2010

Society Contributor:

MATHEMATICAL ASSOCIATION OF AMERICA

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Two decades ago, as Project Kaleidoscope (PKAL) was being launched, undergraduate mathematics was buffeted by outside forces that threatened its quiet comfort zone within academe:

- Demographic and policy pressures that urged collegiate mathematics to change from a social filter to an economic pump;
- Explosion of digital methods in computing that challenged the centuries-old tradition of a calculus-centered mathematics curriculum;
- Powerful calculators and "personal" computers that could readily solve typical textbook math problems;
- The publication of goals for school mathematics under the unprecedented banner of national "standards."

These forces soon nudged undergraduate mathematics to the cusp of major transitions whose consequences are still unfolding.

In 1999, at the tenth anniversary of PKAL, collegiate mathematicians identified the following three changes in undergraduate mathematics as the most significant of the previous decade:

- The increasing realization that "how mathematics is taught is as important as what is taught"—a "signal development" whose consequences "will be reshaping instructional practices in the mathematical sciences for years to come."
- The movement to reform calculus, which "fostered increased interest and intense discussions among mathematicians" and led collegiate mathematics to be "viewed as a leader among STEM disciplines" in revitalization of undergraduate education.
- The use of electronic technology that enables students to "explore and discover mathematical phenomena," facilitates "undergraduate research," connects faculty and students in "virtual communities," and provides "unprecedented access to information."

These three changes were direct consequences of three of the four transitions begun ten years earlier; issues surrounding them are still high on the agenda of most departments of mathematics. The fourth transition—nation-wide standards for school mathematics—took much longer to have any impact on collegiate mathematics, but its consequences are significant and by now often controversial. In addition, emerging areas such as undergraduate research, mathematical biology, and quantitative literacy offer significant new opportunities for mathematics departments to improve the way they educate students. Responses to these changes have noticeably altered the culture of undergraduate mathematics, so that is where we start this retrospective.

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Culture

Since 1990 the response of the mathematical community to the challenges of demographics, digital methods, and computing technology have produced a distinct broadening of professional culture. Instead of limiting their educational focus to the specific courses they are teaching, mathematicians increasingly work collectively on curricular or technology projects. Special interest groups (SIGs) for education-related issues, non-existent twenty years ago, are now routine parts of professional society meetings. The scope of the challenges facing the discipline requires such cooperation, while technology—especially the Internet—makes joint action possible.

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One consequence has been a substantial growth and distribution of the capacity for leadership within the community of collegiate mathematicians, evidenced most readily by individuals involved in PKAL's F21 (Faculty for the 21st Century) program and MAA's Project NExT (New Experiences in Teaching). These programs provide national support networks to junior faculty around issues of curriculum and instruction. As a strategy for preparing the next generation of leaders within undergraduate mathematics, nothing comes close to the success of Project NExT. Early participants in these projects are now moving into leadership positions on their campuses and in professional organizations, thereby spreading even more broadly the benefits of engaged, connected communities.

Another consequence has been a growing recognition of the need for new arrangements for the spaces in which instruction takes place. Interactive pedagogy requires classrooms, laboratories, libraries, and lounges that encourage conversation among and between students and faculty. Ubiquitous technology such as laptop computers and networked cell phones offer opportunities for feedback and group projects that are changing mathematics pedagogy in institutions large and small.

Calculus

Issues associated with calculus are still among the most prominent challenges facing undergraduate mathematics. Today the issue is not competition from discrete mathematics and computer science. Instead, it is the extraordinary growth of AP calculus that has made calculus, de facto, a high school subject. More students now take Calculus I in high school than in college, including a large majority of students who plan careers in STEM-related fields. This change has created myriad problems, not only for departments of mathematics but also for the nation's commitment to effective STEM education.

Despite its popularity with parents and students, most collegiate mathematicians view the unrestrained growth of calculus in high school as quite problematic. One consequence is that many of the nation's most able students never take mathematics in college. Many others rush into AP calculus before mastering the full range of high school mathematics, and then "fail" college placement exams that still demand robust pre-calculus preparation. Still others repeat calculus in college, thereby discouraging classmates who are learning it for the first time, creating significant hurdles for even the most creative instructor, and often boring themselves and winding up uninterested in any further mathematics.

Some worry that Calculus I in college is going the way of College Algebra, being taught primarily to students who took it in high school but did not learn it well, and then repeat it in college (only faster). Calculus II is also profoundly affected by this change since, in contrast to twenty years ago, most students enter it from very different and uneven Calculus I experiences. Even as departments struggle to cope with these unwelcome disruptions to the traditional portal of college mathematics, they are also faced with the long-standing issue of remediation—the large numbers of students who enter college unprepared for either Calculus I or College Algebra.

These challenges, none of their own making, pose serious difficulties for departments of mathematics. Fortunately, there is now widespread recognition of these problems among college mathematicians with many projects, presentations, workshops, and papers offering data and exploring various approaches. These issues are ideal subjects for the emerging culture of self-reflection about pedagogy and curriculum within the mathematical community.

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Technology

When PKAL was founded, calculators and computers commonly used by students were capable only of calculating and graphing. Today most also do symbolic algebra—including all the techniques taught to students in grades 9-14—and much beyond that as well. This new power has changed significantly the way mathematics is used and, to a lesser extent, taught. It has not, however, made as much of a change as it probably should in what mathematics is taught. The revolutionary aspects of this technology to significantly change the curriculum in algebra, calculus, and differential equations (e.g., Wolfram Alpha) have yet to be worked out.

The power of today's computers—indeed, of any new application—is typically revealed in research and industry well before it seeps into the undergraduate curriculum. The mathematical power of computers has supported the growth of fields such as bioinformatics, climate modeling, data mining, genomics, and international finance. College and universities add courses in areas such as these from the top down—first in graduate programs, then as upper division electives, then, occasionally, as first year seminars. Some of these courses are offered by mathematicians, sometimes in their own departments and sometimes elsewhere. As compared with two decades ago, undergraduate curricula in departments of mathematics show definite signs of responsiveness to these new trends.

Nonetheless, for reasons that are more economic than intellectual, the dominant educational impact of technology's new mathematical power is, today, to facilitate online delivery of courses and enable automated homework systems in which the computer's ability to parse and "understand" algebraic manipulations is enlisted in the task of correcting student homework and quizzes. Such systems are now widely used in large universities where student demand is high and budgets for human paper graders are low.

More broadly, politicians and education policy wonks are looking to substitute on-line courses (with automated homework checking) for expensive instructor-led classroom experiences. As news aggregators are replacing newspapers, so "course aggregators" may replace colleges as the organizer and deliverer of mathematics curricula for many students. Whether mathematics can be learned as successfully by this means as in traditional formats is a question no one can answer at this time.

Undergraduate Research

Many mathematicians would argue that the most important innovation in the education of mathematics majors during the last twenty years has been the acceptance and growth of research by undergraduates. Natural scientists have long recognized the importance of inquiry as a strategy to create motivation and enhance learning. Indeed, investigative pedagogy for all students is one of Project Kaleidoscope's founding principles. For years, mathematicians resisted this argument, arguing that "real" research in mathematics required a systematic foundation of undergraduate and graduate courses.

During the last two decades, mathematicians' attitudes have largely changed. Most now understand that with carefully chosen research problems, a supportive community, and artful nudging from advisors, undergraduates can discover new mathematics. This can be an eye-opening experience for both student and advisor. Although many able students are attracted to mathematics by the intrinsic reward of solving hard problems in textbooks or competitions, the exhilaration that comes from discovering mathematics is incomparably greater.

Evidence for the spread of undergraduate research in mathematics can be seen in the increasing number of undergraduates attending and presenting at both regional and national meetings of mathematicians; the inclusion of undergraduate research in various mathematics programs at NSF; and the growth and variety of summer REU programs (Research Experiences for Undergraduates) in colleges and universities of all sizes and kinds. Further evidence for the change in mathematicians' attitudes towards undergraduate research can be seen in the request by many graduate programs for evidence of applicants' research.

As mathematicians began to recognize the effectiveness of inquiry and research as a means of motivating students to persist in STEM disciplines, many began actively seeking topics and problems consistent with this goal. This has led to a significant increase in proxy experiences (e.g., investigative projects, student presentations, capstone experiences) as a part of regular mathematics courses. First year seminars and special projects designed to attract beginning students are now common features in collegiate mathematics programs. Some have even extended the opportunities for research participation to promising high school students in nearby districts.

Mathematical Biology

The last two decades have seen accelerating efforts to connect undergraduate mathematics to other disciplines, especially to biology which itself has become increasingly mathematicized. Many colleges have made substantial changes in their undergraduate mathematics curriculum to better articulate with the needs of the life sciences. These changes often resonate also with other client disciplines (e.g., economics, sociology, business, computer science) since they introduce a more diverse and robust set of mathematical tools than those honed a century earlier for the mathematical needs of physics. Molecular biology and genomics lead directly to widely applicable topics in combinatorics, geometry, and discrete mathematics and open doors to many careers in fields far removed from the life sciences.

More generally, the widespread availability of technology-mediated data has increased significantly the role and value of mathematics and statistics in the natural, social, and behavioral sciences. Gradually, college mathematicians have begun to recognize the importance of substantive engagement with other disciplines. Although the motivation for this new openness has sometimes been mere self-preservation—students interested in mathematics often found other majors more interesting than the traditional mathematics major—a more generous interpretation is that such engagement offered mathematicians a way to cooperate with their colleagues in other fields in encouraging students to pursue STEM careers. In this respect, mathematics has became a full partner in the "natural science community" that was part of PKAL's original vision.

Evidence of how widespread this cooperation has become can be found, among other places, in the Mathematical Association of America's Curriculum Foundations Project which convened a multi-year series of weekend roundtable discussions between mathematicians and practitioners in other fields to discuss the mathematical needs of different fields. Summaries of these discussions, made available on-line and in published reports, show a striking degree of convergence among the different disciplines in what they want from mathematics. Not surprisingly, many of these themes are also noticeably similar to the "what works" messages expressed by PKAL during these same years.

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Quantitative Literacy

One consequence of our technology-rich environment is a growing reliance on data—whether meaningful or not—as the basis for making personal and policy decisions. Traditional mathematics courses in algebra and calculus taught students virtually nothing about data. One of the more promising movements of the last two decades has been the growing awareness of the importance of quantitative literacy (QL) as a component of liberal education for citizens, a task requiring leadership from mathematicians in cooperation with other disciplines that reason with data.

Exploration of QL has taken many forms, ranging from the increased emphasis on AP statistics in high school to efforts on some campuses to transform College Algebra from a toolkit of manipulative skills to a survey of mathematical models in the context of contemporary problems. The recognition on most campuses that the responsibility for students' quantitative literacy cuts across many departments has been really important, and has opened yet another urgent topic for dialogue between mathematicians and their colleagues in other fields.

As more and more students arrive in college having had a taste of statistics from high school—some through AP, others not—mathematics departments will need to develop appropriate curricular glide paths for students whose preference is for data over abstractions. Professional networks have already sprung up to assist with this challenge: the MAA has a special interest group (SIG) devoted to QL and the National Numeracy Network, a new interdisciplinary professional organization has begun the on-line *Journal Numeracy: Advancing Education in Quantitative Literacy.*

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STEM Education

In recent years, the effort begun in the late 1980s to establish national standards for K-12 mathematics education—the fourth transition mentioned above—has gained renewed momentum as part of national efforts to improve education in STEM disciplines. Productive dialogue between college and university mathematicians and K-12 teachers of mathematics has increased substantially at local, regional, and national levels. Each summer several "vertically integrated" institutes offer opportunities for college faculty, graduate and undergraduate students, K-12 teachers, and high school students to work together on mathematics. As school teachers renew their skills by learning mathematics from college faculty, so college mathematicians have discovered that they too have a lot to learn from those who teach mathematics to younger students.

Much of this new dialogue about teaching and learning mathematics grew out of discussions concerning the direction of educational reform, for example, about the role of calculators in school mathematics; about strategies to reform the way calculus is taught; about the goals, clarity, and rigor of K-12 standards; about the mathematics required of prospective K-12 teachers; about the exams used for high school exit, college entrance, and mathematics placement; and about the role of teaching in faculty evaluations (both in schools and universities). These issues are not new. What is new is that they are now part of open discussion among K-12 teachers, college faculty, and educational administrators (not to mention students, parents, journalists, and politicians).

In the last twenty years professional societies representing college mathematicians have for the first time in their history regularly organized symposia, invited speakers, and published influential books dealing with a variety of education issues. These include the preparation of teachers of mathematics, standards for excellence in college mathematics, strengthening assessment of undergraduate mathematics, strategies for effective teaching, responding to diverse students interests, meeting the mathematical needs of other disciplines, and leading college and university mathematics departments. Professional meetings of university mathematicians, which in the mid-1980s were predominantly devoted to mathematical research and applications, are today a nearly equal mix of mathematics and mathematics education. For a community steeped in a tradition that focused only on research and exposition of mathematics, the very visible emphasis on teaching and learning is a major change in the culture.

Perennial Issues

Among the more vexing issues facing undergraduate mathematics are the seemingly permanent problems of remediation and equity. Since well before PKAL was founded, approximately one in three students who enter higher education need remediation in mathematics (and often also in English). Not surprisingly, these students are much less likely to graduate than those who enter well prepared. The substantial increase in the proportion of students who go to college now as compared with several decades ago no doubt contributes to the continuing need for remediation. Paradoxically, however, although the proportion of students who take advanced high school mathematics has also increased over these same years, the need for remediation in college has not diminished.

Concerning equity we find a different but equally paradoxical situation. Many campus programs have been able to reduce the gap in performance, persistence, and completion between majority students and those from groups that have been historically underrepresented in mathematically based fields. Yet these programs remain local and rare, showing little evidence of national impact. Programs that work tend not to be systemic, depending instead on the personal energy of an inspired local leader. The data show some temporary success in improving the participation of women but virtually no progress in achieving parity among college mathematics graduates from other underrepresented groups.

Emerging Issues

As regional accreditors begin demanding evidence that reliable assessment is used regularly to improve instructional programs, more and more mathematics departments have begun to wrestle with this thorny issue. With support from NSF, the MAA has sponsored national projects and workshops on assessment, and published volumes of case studies. (Indeed, mathematics was one of the first of the liberal arts and sciences to establish a national committee on assessment at the college level.) Nonetheless, assessment remains an idiosyncratic and unpopular issue on most campuses, with the majority of faculty preferring to meet new requirements with minimum effort and least change to the established order. Unlike many other topics on their agenda, assessment is not one that mathematicians seem eager to engage.

A second emerging issue is that the annual production of mathematics majors is insufficient even to supply the nation's secondary schools with well-qualified teachers, not to mention all the other attractive options for which mathematics majors are well qualified. Although high schools are preparing more students than ever with strong mathematics backgrounds, mathematics majors make up a declining proportion of college graduates. So in addition to the ongoing studies of how best to prepare teachers of mathematics, departments also need to figure out how to persuade more qualified students to major in mathematics.

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