High School Mathematics

Introduction to the High School Mathematics Expectations prepared in 2005-07 for the American Diploma Project of Achieve, Inc.

For well over a century Americans have recognized the value of having children learn "the three R's." Even now, in a society transformed by technology, reading, writing, and 'rithmetic constitute the core of high-stakes tests given to students throughout the K-12 system. Literacy and numeracy are essential prerequisites to jobs and learning, so they become schools' most important—but far from only—goal.

Americans are also well aware that we have failed to achieve this number one goal. High dropout rates from high school, high enrollments in remedial college courses, and continuing complaints from employers attest to the magnitude of this failure. Approximately a third of our youth do not finish high school on time, and another third finish with significantly substandard skills. Only a third, predominately white and well off, complete school having met the expectations that states have enunciated for the three R's.

In many districts and states it is the third of the three R's that causes the greatest concern. Whereas in earlier eras only 'rithmetic was needed to prepare for work in factories or farms, today everyone recognizes that much more is needed. But not everyone agrees on just what that "more" consists of. What mathematics, beyond arithmetic, should all high school graduates be required to know and be able to do?

Some recommend the mathematics needed to prepare for college-level mathematics (e.g., calculus); others suggest a mixture of mathematical and statistical literacy necessary for informed citizenship; still others commend a blend of computer and quantitative skills that are expected in most modern workplaces. Depending on the policy context in different states and districts, some want to require all students to take the same core (whatever it may be), while others argue for options (i.e., tracks) in order to maximize student motivation. Added to this cacophony are arguments about pedagogy (e.g., direct instruction vs. inquiry approaches, individual study vs. teamwork), pacing (how early to begin formal algebra), and rigor (routine exercises vs. higher order thinking).

For over a decade, Achieve has been an active partner in these discussions. We have learned to recognize the enormous gap between our ideal (all students prepared for college and work) and current reality (one third drop-outs, one-third needing remediation, one-third prepared). Moreover, we have learned to appreciate the special importance of mathematics as a factor in creating that gap and as a potential tool for helping close it.

What mathematics really matters?

The mathematics needed for work, for college, and for citizenship has a common trunk supporting many distinct but interlocked branches. The trunk includes algebra (symbols),

geometry (shapes), and statistics (data)[*]. All three are equally important. Significantly, they intertwine with each other, thereby strengthening the trunk of mathematics from which its many branches and applications grow. Algebra builds on geometry; statistics relies on algebra; geometry employs data and algebra. In a well structured curriculum each reinforces the other.

To ensure that students are prepared for whatever life brings them—including demands of active citizenship, technology-enriched employment, and postsecondary education—all students should study a common core of algebra, geometry, and statistics. For most students, mastery sufficient for life after high school will require three years (3 Carnegie units) that build on a substantial elementary school program focused on the fundamentals of arithmetic (including standard algorithms, fractions, percents, ratios, and proportions).

Typical students require seven or eight years to attain sufficient mastery of arithmetic to be ready for high school mathematics. Compressed curricula that make children move on before they are truly ready risk shallow learning and swift forgetting. There is no reason to rush the learning of mathematics. The failure in the education system that we need to overcome is not lack of speed but lack of long-term learning.

Thus for almost all students, the desired three year core curriculum in high school mathematics can take place in grades 8-10 or 9-11, or for students needing more time, in grades 8-11. The goal should be to complete a balanced core (algebra, geometry, statistics) in time for students to take required state or district exams at the end of 11th grade. Whether this core is organized in separate courses or through an integrated curriculum is immaterial. Equally irrelevant is whether it is focused largely on mathematics in the abstract or on the uses of mathematics in the world around us. What matters is that the mathematics that the students learn is sound and substantial. The goal is to ensure that all students master a broad foundation by the end of 11th grade.

This leaves the 12th grade available for a variety of electives, including additional algebra, geometry, or statistics, as well as a variety of courses in mathematical modeling and applications of mathematics. Students who complete the core in grades 8-10 could take a precalculus course in grade 11 and calculus in grade 12. The majority of college-bound students would be better advised to take precalculus in grade 12 and leave calculus for college where it is a mainstream entry course. Too many students who take calculus in high school now wind up placing into high school-level mathematics courses in college because their rushed learning doesn't last.

Finally, we note that for students to learn mathematics well, they need to use it. To make this both possible and certain, high schools need to coordinate their mathematics curriculum with the uses of mathematics in parallel courses in the social and natural sciences and in vocational programs. To help our nation achieve its educational goals, teachers of subjects in which mathematics can be used should ensure that it is actually used. That is the best way to make sure that the mathematics learned in mathematics class is wisely invested rather than immediately forgotten.

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[*] Algebra is about the use symbols to represent linear, exponential, quadratic, and logarithmic phenomena; geometry is the study of shapes, including deductive (Euclidean) and empirical (measurement) approaches in two and three dimensions; statistics is about the use of data in various contexts (e.g., probability and risk, social choice (voting), fair division, correlation and causation, interpolation and projection).