Refractions, Reflections, Recombinations: Democratising Maths for Mass Education

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Abstract: Two powerful currents affect collegiate mathematics at the beginning of the 21st century: the growing world-wide emphasis on universal access to postsecondary education and the extraordinary spread of mathematical methods to fields as diverse as cinema and genomics. As the importance of mathematics grows and its utility expands, so does competition for mathematically-minded students. To thrive, mathematics must diversify and democratise, allowing students access to its unique and powerful benefits from many different angles.

In 1998 UNESCO convened a World Conference on Higher Education to establish a vision and action agenda for higher education in the twenty-first century. One result of this meeting was the recognition and endorsement of what has been called "massification" of higher education—its transformation from an opportunity for the elites of society to an institution intentionally serving a larger and more diverse clientele. The thesis I wish to explore is that at the same time and for many of the same reasons, mathematics has undergone a similar transformation. Moreover, the transformed mathematics and emancipated higher education now exist in a new ecosystem where each depends on the other in order to achieve its goals.

To illustrate this argument, I want to look through a mathematical lens at the opening paragraphs of the World Declaration on Higher Education that emerged from UNESCO's meeting in October, 1998.[1] It begins with a sweeping finding that applies to mathematics as well as it to higher education:

> At the turn of a new century, we find unprecedented demand for and great diversification in higher education [*think* "mathematics"] as well as increased awareness of its vital importance for building the future, for which younger generations need to be equipped.

The UNESCO statement goes on to analyse challenges of higher education in terms that echo perfectly the challenges facing mathematics. Here's the original, slightly abridged; as you read, think "mathematics" wherever "higher education" appears:

> Higher education encompasses all types of studies, training, and research at the postsecondary level, yet everywhere it is faced with great challenges related to financing, equity and access, staff development, enhancement of quality in teaching and research, relevance of programs, and employability of graduates. At the same time, higher

> education is also being challenged by new opportunities relating to technologies that are improving the ways in which knowledge is produced, managed, disseminated, accessed and controlled.
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> The second half of the twentieth century will go down in the history of higher education as the period of its most spectacular expansion. But it is also the period which has seen the gap regarding access for higher learning, already enormous, become even wider. It has also been a period of increased socio-economic stratification in educational opportunity within countries, including in some of the most developed and wealthiest nations. Without adequate higher education providing a critical mass of skilled and educated people, no country can ensure genuine endogenous and sustainable development.
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> Higher education has given ample proof of its viability over the centuries and of its ability to change and to induce change and progress in society. Owing to the scope and pace of change, society has become increasingly knowledge-based so that higher learning and research now act as essential components of sustainable development of individuals, communities, and nations. Higher education itself is confronted therefore with formidable challenges and must proceed to the most radical change and renewal it has ever been required to undertake.

It is truly amazing how well UNESCO's description of higher education fits what we all know so well about mathematics. Expanding uses are gradually changing mathematics from a subject for elites to a subject for the masses; technology is changing the way mathematics is practiced; information-age challenges are expanding the frontiers of mathematics at an unprecedented rate; and finally, gaps between those with and without mathematical expertise both reflect and exacerbate socio-economic divides in society. As we will see, the confluence of these challenges creates what U.S. readers might well call "a perfect storm" (after a popular book and movie of that title about a North Atlantic storm created by the rare convergence of three weather systems). Indeed, university mathematics is not unlike a small boat tossed around by the heavy seas of change. In what follows, I offer a sampling of evidence for the nature and magnitude of forces converging on undergraduate mathematics and suggest how, with imagination and will, university mathematics departments can align these forces to their and society's benefit.

Democratisation of Postsecondary Education

The 1998 first World Conference on Higher Education adopted "education for all" as a guiding theme and stressed the increasing importance of postsecondary education for the knowledge-based economy. Since then higher education has expanded quite rapidly in many countries, especially those with emerging global economies. In China, for example, between 1998 and 2002 total enrolment in higher education more than doubled, from 6.4 million to 15.1 million. (For comparison, it took the United States 35 years—from 1965 to 2000—to achieve similar growth.) Between 1993 and 2000, higher education enrolments in India increased from 6.2 to 9.3 million, and in Korea enrolments increased from 2.9 to 3.5 million between 1998 and 2001—slower than in China or India, but still substantial in annual terms.[2] Similar patterns of substantial recent growth have occurred in countries of central and eastern Europe where higher education institutions, both public and private, have responded to increased demand for postsecondary education.

According to UNESCO, much of this growth has been made possible by significant expansion in private higher education and distance learning. Malaysia is an extreme example of the former, where the number of private postsecondary institutions has increased seven-fold in the last five years, from approximately 100 to nearly 700. Both in the East and the West, new private institutions of higher education, often specialized, have contributed significantly to the capacity for growth. Not surprisingly, rapid growth has created problems of educational quality and accountability.

It is important to note that the general pattern of increased access to higher education has significant and disturbing exceptions, primarily in sub-Saharan Africa where enrolment ratios are the lowest of all the regions of the world. In 2003, a UNESCO study concluded that "in order to meet human resources needs, several countries in this region would need to triple their current higher education enrolments by 2010."[3] Many other countries, besieged by war, disease, or famine, have been unable to respond to UNESCO's 1998 call for "education for all."

Aggregate numbers measuring increased enrolments mask significant differences in educational patterns both among and within nations. In some countries that are otherwise quite different (for example, Malaysia and the United States) women make up a significant majority of postsecondary students whereas in other nations (for example, India and Korea) men outnumber women by about 2:1. UNESCO data also show that the balance of enrolments varies significantly among disciplines, both within and between countries. In some nations, the natural sciences and engineering comprise between a third and a half of all bachelor's degrees, whereas in other countries the figure is not much more than 10%.

Most countries also exhibit significant differences in educational attainment among different ethnic groups. In the United States, for instance, our African-American and Hispanic sub-populations attain university degrees at only half the rate of the white European majority.[4] How best to decrease this "gap" (as it has come to be called) is one of the most contentious issues in American higher education. From what little I know about New Zealand, I gather that you have similar concerns about gaps in educational attainment, especially at the tertiary level, between the Maori and Pacific Islander minorities and the European majority.

Because it is a central school subject of increasing importance in our digital age, mathematics plays a specially strong role both in perpetuating these ethnic gaps and in exacerbating their effects. For social and economic reasons--to reduce inequity and to enhance industry--many countries have established aggressive goals to increase the proportion of enrolments in science and technology programs, especially among those who have been traditionally underrepresented. One of the most important strategies is distance learning; another is new flexible degree programs; a third is the development of new colleges, often private. All these strategies enhance access and enable expansion of higher education. They all also challenge the structure of traditional mathematics as traditionally delivered.

A second major theme cited by the UNESCO report is the increasing participation in higher education of older ("mature-age") students and the related recognition of the value of life-long learning and periodic professional development. In the United States, the average age of students in community colleges—which account for over half of postsecondary mathematics enrolments—is over 30. These changes increase pressure to regularize transfer of credit among different institutions and to recognize the growing number of certificates, qualifications, and degrees. "To meet the needs of an increasingly globalised workforce, professionals need qualifications that are portable as well as possessing relevant cultural and linguistics skills."[5]

University mathematics is profoundly affected by all these changes. Growth in enrolment brings not only more students, but students with different motivations, more varied preparation, and more diverse career goals. Since the canonical curriculum in mathematics does not serve well the needs of many of these new students (for example, relatively few need calculus or differential equations, but many need to understand the normal distribution and create spreadsheets), mathematics departments need to develop courses and programs suitable for a greater variety of student interests. The growing mismatch between mathematics departments' traditional missions and these newer demands frequently creates stress both on students and faculty as well as between mathematicians and their colleagues in other parts of the university. The worldwide emphasis on democratising higher education requires that mathematicians envision something never before contemplated: higher mathematics education for all.

Diversification of Mathematics

A second contribution to the "perfect storm" is the dramatic diversification of mathematics itself. (I seem to have let my storm metaphor lead me away from the more mathematical images in my title: refractions, reflections, recombinations. But in this section they come together, since the forces that cause mathematics to diversify are in large measure created by repeated reflection of mathematical methods from one area of application to another. As the recombination of DNA helps sustain species' strength, so the recombination of mathematical tools brought about by challenges of new applications breathes new life and increased power into the ever-expanding discipline of mathematics.)

The expansion of mathematical methods into such diverse areas as genetics, finance, and even cinema has significantly changed the balance of mathematical theory and application. It has also significantly increased the number of people whose careers and lives are touched by, or depend on, mathematics.   Evidence for this is not hard to find. For example, in recent years the American Mathematical Society (AMS) has published books on topics as diverse as immunology, finance, and the mathematics of sport. Other publications—just from AMS—have dealt with the relations of mathematics to:

Bioconsensus	Folding Objects	Mobile Networks	Reliability
Computer Security	Genetics	Natural Resources	Robotics
Cryptography	Growing Crystals	Physiology	Simulation
DNA Computers	Locating Facilities	Protein Folding	Switching Networks
Ecology	Market Forces	Psychophysiology	Tomography
Fair Allocation	Medical Imaging	Records	Vision

This catalogue is worlds apart from mathematics' traditional triumvirate of algebra, geometry, and analysis, and even from mathematics' historic areas of application in engineering and the physical sciences.

The expanded role of mathematics in society is also attested to in an international report prepared for the U.S. National Science Board (NSB) as part of its responsibilities to monitor quality and direction of scientific research in the United States.[6] This influential report describes the impact of mathematics on society as "pervasive" and notes that mathematics itself is evolving in response to problems posed by science, government (defence, security), business, and technology. They cite as examples of mathematics' support for societal needs such far-flung areas as automated manufacturing, electronic security, analysis of the human genome, and valuation of options.

Medicine offers a good example of the pervasive character of mathematics in modern society. Basic research on genomes (of humans as well as of pathogens) depends on mathematical tools to search databases for matching patterns. Pharmaceutical research depends on computational geometry to analyse the folding of proteins to identify potential sites for drug interaction. Medical imaging systems (CAT scanners, magnetic resonance imaging (MRI), nuclear imagers) depend on the mathematical processing of signals. And increasingly, management of financial and patient records requires complex algorithms to design secure and efficient databases.

Another example cited in the NSB report is environmental monitoring, which depends heavily on mathematical models of oceanic and atmospheric conditions. Climate models require the manipulation of massive quantities of data and the study of complex simultaneous interactions. All such models involve uncertainty; assessing these models' validity also requires mathematics. Obviously, accurate projection of the impact of human activities on the environment is essential to the formulation of sound public policies.

At the same time as mathematics has been penetrating far-flung corners of academia, demand for information about mathematics from laypersons has grown enormously. From a time in the 1960s when there were only 5-10 popular books about mathematics published each year, now there are four or five published each month. These trade books satisfy a strong public thirst for insight into the seductive yet apparently impenetrable mysteries of mathematics.

It is entirely natural that as people encounter mathematics in the news and on the job, however fleetingly, many find renewed interest in a subject whose last classroom contact left them with distinctly unpleasant memories. With enough distance from their own experiences of school or college mathematics, natural curiosity returns (although guarded with wariness about special sources of bad memories, typically formulas or "word problems"). This renewed public interest is sufficient to sustain a niche market in popular books about mathematics that would have been unthinkable three or four decades ago. Some varied recent examples of this genre are:

Damned Lies and Statistics by Joel Best. (University of California, 2001).
How the Universe Got Its Spots by Janna Levin. (Princeton, 2002).
Fragments of Infinity: A Kaleidoscope of Math and Art by Ivars Peterson. (Wiley, 2001).
Mathematics and the Roots of Postmodern Thought by Vladimir Tasic. (Oxford, 2001).
The Mathematics of Juggling by Burkard Polster. (Springer, 2002).
What Are the Odds? The Chances of Extraordinary Events in Everyday Life by Jefferson Hane Weaver. (Prometheus, 2002).
The Music of the Primes by Marcus Du Sautoy. (Harper Collins, 2003).
Gamma: Exploring Euler's Constant by Julian Havil. (Princeton, 2003).
Imagining Numbers by Barry Mazur. (Farrar, Straus and Giroux, 2003).

Mathematicians take for granted what Eugene Wigner called the "unreasonable effectiveness" of mathematics in many domains of life and learning. Even though they may not know much about demography or genomics, the fact that such subjects depend on mathematics comes as no surprise to mathematicians. But to a public who views mathematics as an esoteric, sclerotic subject far distant from the real world, these connections are not common knowledge. Gradually popular exposition will enable the public to see what mathematicians know—that mathematics is an enabling discipline that pervades nearly every domain of human experience.

Diversification of Mathematical Practice

Here I wish to distinguish between *mathematics* and mathematical *practice*. Mathematics, the subject of the previous section, is an ancient but still very active discipline whose frontiers are constantly expanding as a result of its increasing engagement with problems of the modern world. Mathematical practice refers to the multitude ways in which mathematics is practiced by people in their routine work and daily lives. Since much of this practice is performed as a routine part of some other activity, it is rarely thought of as "mathematics."[7]

In today's information age, economic prosperity depends on "working smarter," not just harder. In the United States, businesses regularly complain about deficiencies in the technical and problem-solving skills of their workers. Indeed, the cost of finding skilled employees has become a serious drag on many sectors of the U.S. economy. (As a consequence, at least until very recently, the U.S. recruited many technically skilled workers from other countries.) By and large, the most severe shortages occur in jobs that require some education beyond secondary school, but not nearly as much as a bachelor's or master's degree. In the United States, we call this labour sector the *technical workforce*.

Increasingly, jobs available to those with just one or two years of college education require mathematical thinking.[8] Many corporations worldwide are moving to a flat management structure in which workers are responsible for their own quality control and management through small employee-led teams. Such workers no longer have the luxury of just doing what they are told. They are now expected to develop their own problem-solving strategies, often using spreadsheet, statistical, or mathematical tools suitable to a data-rich high-technology work environment.

Technical careers are found in all areas of the economy—from agriculture to nursing, manufacturing to marketing. Technical employees are often expected to use mathematics, but generally in situations quite different from the professional-level challenges that confront advanced degree holders. The mathematics needed in technical jobs typically involves specialized use of elementary mathematics where the required reasoning is nonetheless cognitively sophisticated. Typically, issues arising in a high-performance work environment require multi-step solutions to open-ended problems never before encountered. (If they had been encountered and solved before, they would no longer be problematic.) Emphasis is often on estimation, measurement, tolerances, and conversions—not just on calculation. Computer tools, mostly spreadsheets, are common.

Prospective technical employees do not usually need calculus or even advanced algebra, but a plethora of more useful mathematical skills that are insufficiently emphasized in secondary education. Examples include statistics and three-dimensional geometry, systems thinking and estimation skills, and a capacity to interpret and present technical information. In addition to procedural skills, these technical employees need sufficient theoretical understanding to recognize and deal with situations when something goes wrong.

Much of the technical work force is trained in what has been called a "parallel universe" of postsecondary education—on-line courses, for-profit colleges, certificate programs, and virtual universities.[9] As these new institutions compete with traditional universities for enrolment and resources, universities respond, depending on their mission, with diversified programs of their own. Hotel management, forestry, and hazardous waste disposal now appear alongside economics, genetics, and molecular biology on lists of programs offered by major universities. Students in all these programs take courses in mathematics—but not the same old courses, or even the same new ones.

Increased demand for mathematical practice associated with jobs that previously needed no more than simple arithmetic is yet another force contributing to the democratisation of university mathematics. In addition to preparing professionals for careers in mathematically intensive fields, mathematics departments must now also provide suitable preparation for the future technical workforce. Because different jobs require different mathematics, because many prospective students for these jobs arrive at universities without strong preparation in mathematics, and because these jobs are often growing faster than higher-level careers, the task of preparing the technical workforce makes enormous and unprecedented demands on departments of mathematics. To respond effectively and to serve students well, mathematics departments need to work closely with other departments and programs, many of which have not normally been seen as natural partners for mathematics departments.

One consequence of such cooperation is the discovery that to practitioners, mathematics is more inclusive than the subject taught in mathematics departments. Those who use mathematical and quantitative tools in their routine work generally include under the label "mathematics" all that mathematics departments normally teach (algebra, geometry, probability, etc.) plus mathematical modelling, statistics, computing, spreadsheets, statistical packages, and various numerical and symbolic manipulation packages. Theirs is a "big tent" view of mathematics—to borrow a phrase from American politics—which, unfortunately, is not always the view within mathematics departments.

From society's perspective, preparing a robust, well-trained technical workforce is every bit as important as preparing a strong scientific workforce. Both require mathematics, but the former involves by far the larger number of students.

Mathematics for Everyone

The motto "education for all" of the World Conference on Higher Education is not only about employment skills, but also about the need for educated citizens who will make well-informed decisions about their own lives and in their contributions to societal decisions. These days most such decisions are about issues expressed in terms of numbers, graphs, and other quantitative data. News media use graphs and numerical tables to explain municipal budgets, investment options, health risks, and environmental policies. Global warming, trade agreements, agricultural subsidies, and spread of diseases are all monitored with quantitative evidence. In daily life, individuals must sift through confusing numerical claims about costs of basic services such as transportation, banking, and telephone contracts. Political debates, legislation, and court decisions—the heart of civic participation—all depend increasingly on quantitative analyses.

The quantitative skills and habits of mind required to be a good citizen and responsible adult are typically called numeracy or quantitative literacy. Despite very broad and powerful definitions (for example, "the ability and inclination to use mathematics effectively—at home, at work and in the community" [10]), in most nations numeracy is identified with elementary education as parallel to basic literacy. This is a mistake.

The mathematical principles underlying everyday quantitative challenges are generally elementary: for informed citizenship, ratios are far more important than integrals, triangles more than groups. Nonetheless, the contextual embedding of quantitative problems arising in modern society creates situations that are quite subtle, often surpassing the ability of most adults—including many who are well trained in traditional mathematics.[11] For a variety of reasons, quantitative literacy has received little attention in either secondary or postsecondary curricula. But the recent rapid escalation of quantification in society has made this inattention a serious problem. Correcting this neglect is an important new responsibility for university departments of mathematics.[12] Quantitative literacy, argues a senior U.S. labour economist, is about the "democratisation of mathematics."[13]

Numeracy is not the same as mathematics; it is certainly not just elementary mathematics, as some aver. For example, the increasing role of abstract index numbers (for example, stock market averages) and fragile budget projections in individual and public decisions suggests an alarming possibility: we may no longer understand the language we speak. As mathematics undergirds many traditional trades and professions, so it also made possible the rise of the "culture of finance" that in turn made possible the accounting scandals that have undermined economies the world over.[14] Being unconstrained by reality, numbers that represent mathematical abstractions (for example, weighted indices, IQ ratios, Black-Scholes risk values), can behave differently from numbers that represent real objects or events—which are the only numbers students have experience with in their elementary mathematics courses. College level experience with mathematical abstractions is crucially important to make a person numerate.

On the other hand, because quantitative literacy depends so deeply on context, mathematics departments should never be fully responsible for students' numeracy. Helping students learn appropriate uses of quantitative tools is the responsibility of every department, each in its own domain. Yet without leadership from mathematics departments, quantitative literacy is unlikely to become an effective part of college students' education. Educating for quantitative literacy is actually very difficult—almost certainly more difficult than teaching calculus (at least as it has traditionally been taught). Since quantitative literacy is not an academic discipline, it has no natural departmental advocate; since it is necessarily contextual, it spans all disciplines. Transference of skills is the most important test of understanding, but it is a test that we all know is very difficult to meet. The best strategy to achieve this goal is mutual reinforcement of methods and concepts in a variety of contexts. That can only be achieved by an intentional plan that engages students' minds "from all angles"—or at least from many different angles.

Although daunting in many respects, the challenge of numeracy—of "mathematics for all"—provides a real opportunity for university mathematics departments not only to meet an important and growing responsibility but also to increase support for mathematics. All over the world, government leaders are increasing expectations that universities serve broad public purposes, not merely elite professional interests. This demand arises both from increasing democratisation of governments (making them somewhat responsive to the people they govern) and from increases in public funds supporting higher education. Accountability pressures on mathematics departments are now external, not just internal; in the United States, at least, they arise as often from politicians as from peers. Mathematics departments that exercise effective leadership in support of every students' quantitative literacy can meet a pressing public need while at the same time engage in challenging interdisciplinary issues that can advance both research and pedagogy.

There are many ways to approach this task, none of them easy or clearly superior to others. Some institutions debate balkanisation—whether separate programs for separate branches of the mathematical sciences can better meet the quantitative and mathematical needs of all students. Others seek to integrate introductory university mathematics with other subjects into "core curricula" that transcend the boundaries that separate traditional disciplines. Some create university-wide expectations for demonstrated mathematical or quantitative proficiency while others leave such requirements to each particular program and major. Some create special courses in quantitative literacy (or what in the U.S. is cynically called "math for poets") while others seek to embed mathematical methods across the curriculum. Regardless of the strategy, what will determine success or failure is the degree of commitment and leadership shown by mathematics faculty to the emerging agenda of "mathematics for all."

Refractions, Reflections, Recombinations

The evidence presented here, only a tiny fraction of what might be assembled, suggests how the forces that combine to produce our "perfect storm" flow from recombination and reflection of several inexorable movements, notably the democratisation of societies, the digitisation of knowledge, and the diversification of mathematics.

- Democratic cultures require "education for all," thus higher education for many. Since mathematical knowledge enhances power, in a democracy mathematics must be shared widely.
- Digitisation of knowledge—in the arts, crafts, sciences, and trades—has spread the influence of mathematics to all human endeavours. Thus nearly all students need postsecondary mathematics to support their major programs or career aspirations.
- Diversification of mathematics, prompted by challenges of the information age, creates opportunities and attractions for everyone: for all, quantitative literacy; for many, technical training; and for some, pre-professional specialization.

These movements reflect off each other and recombine to create new forces, some positive, some not. Democratisation is enhanced by digital communication: dictators can no longer maintain total control on information. Yet so too are interlocked financial empires and militants' global networks. Digitisation of knowledge enables the global economy: distance hardly matters anymore in transmission of ideas and goods. But this same capability, based on mathematical algorithms, also strengthens weapons of terror and mass destruction. Diversification of mathematics empowers citizens on whom democratic societies depend: ordinary people can now analyse and challenge government data. At the same time, the spreading importance of mathematics accentuates the personal consequences of persistent gaps in mathematical knowledge that both reflect and reinforce socio-economic strata.

Moving in waves, here slowly and there rapidly, these interconnected forces are relentlessly transforming mathematics from a focused subject for the few to a broad-based subject for the many. For the benefit of society and for the good of the students we educate, it is imperative that mathematicians embrace these broader, trans-disciplinary responsibilities.

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